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Renormalization Group and Effective Field Theory Approaches to Many-Body Systems

 Springer

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Preface

Over the past years, there have been many important developments based on effective field theory and the renormalization group in atomic, condensed matter, nuclear and high-energy physics. These powerful and versatile methods offer novel approaches to study complex and strongly interacting many-body systems in a controlled manner.

These Springer Lecture Notes in Physics combine selected introductory and interdisciplinary presentations focused on recent applications of effective field theory and the renormalization group to many-body problems in

- atomic physics,
Jean-Paul Blaizot: Nonperturbative Renormalization Group and Bose-Einstein Condensation.
- condensed matter physics,
Bertrand Delamotte: An Introduction to the Nonperturbative Renormalization Group.
- nuclear physics,
Richard Furnstahl: Effective Field Theory for Density Functional Theory,
Thomas Schaefer: Effective Theories of Dense and Very Dense Matter,
Bengt Friman, Kai Hebeler and Achim Schwenk: Renormalization Group and Fermi Liquid Theory for Many-Nucleon Systems.
- and high-energy physics,
Holger Gies: Introduction to the Functional Renormalization Group and Applications to Gauge Theories.

The discussions of these Lecture Notes are aimed at graduate students and junior researchers, and hopefully offer an opportunity to explore physics across subfield boundaries at an early stage in their career.

We would like to thank Jean-Paul Blaizot and Wolfram Weise for their encouragement with this volume, and Christian Caron for his kind help with putting the volume together.

Darmstadt and Strasbourg, September 2011

Janos Polonyi
Achim Schwenk

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