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Lectures on Gaussian Processes

Springer

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ISSN 2191-8198 e-ISSN 2191-8201
ISBN 978-3-642-24938-9 e-ISBN 978-3-642-24939-6
DOI 10.1007/978-3-642-24939-6
Springer Heidelberg Dordrecht London New York

Library of Congress Control: 2011943344

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

Gaussian processes can be viewed as a far-reaching infinite-dimensional extension of classical normal random variables. Their theory is one of the most advanced fields in the probability science and presents a powerful range of tools for probabilistic modelling in various academic and technical domains such as Statistics, Forecasting, Finance, Information Transmission, Machine Learning—to mention just a few.

The objective of these lectures is to present a quick and condensed treatment of the core theory that a reader must understand in order to make his own independent contributions. The primary intended readership are Ph.D/Masters students and researchers working in pure or applied mathematics. The knowledge of basics in measure theory, functional analysis, and, of course, probability, is required for successful reading.

The first chapters introduce essentials of the classical theory of Gaussian processes and measures. The core notions of Gaussian measure, reproducing kernel, integral representation, isoperimetric property, large deviation principle are explained and illustrated by numerous thoroughly chosen examples. This part mainly follows my book "Gaussian Random Functions" but the chosen exposition style is different. The brevity being a priority for teaching and learning purposes, certain technical details and proofs are omitted, rendering approach less formal, more appropriate to the lecture notes than to a textbook.

Obviously, new issues that emerged during last decade are also present in the exposition. Inequalities related to correlation conjecture and to other extremal problems, the entropy approaches to evaluation of small deviation probabilities, expansions of Gaussian vectors, relations to the theory of linear operators, and links to quantization problems for random processes fit into this category.

The short lecture notes by no means aim to provide a complete account of immense research field in pure and applied mathematics related to Gaussian processes. A few indications on further possible reading are given in "Invitation to Further Reading".

In university teaching, one can build a one-semester advanced course upon these lectures. Such courses were given by the author in Russia (St. Petersburg State University), in France (Université Lille I), in Germany (TU Darmstadt), in Finland (Helsinki University of Technology) and in USA (Georgia Institute of Technology) during last years. I am grateful to all mentioned host institutions for opportunity to teach my favorite subject in their rooms.

My sincere thanks go to Armin Straub for taking enthusiastic notes which served as an early draft of this text, and to Alexei Khartov for careful reading of the manuscript.

Abstract

Gaussian processes can be viewed as a far-reaching infinite-dimensional extension of classical normal random variables. Their theory presents a powerful range of tools for probabilistic modelling in various academic and technical domains such as Statistics, Forecasting, Finance, Information Transmission, Machine Learning—to mention just a few. The objective of these Briefs is to present a quick and condensed treatment of the core theory that a reader must understand in order to make his own independent contributions. The primary intended readership are Ph.D./Masters students and researchers working in pure or applied mathematics. The first chapters introduce essentials of the classical theory of Gaussian processes and measures with the core notions of reproducing kernel, integral representation, isoperimetric property, large deviation principle. The brevity being a priority for teaching and learning purposes, certain technical details and proofs are omitted. The later chapters touch important recent issues not sufficiently reflected in the literature, such as small deviations, expansions, and quantization of processes. In university teaching, one can build a one-semester advanced course upon these Briefs.

Keywords Gaussian processes · Gaussian measures · Isoperimetric inequalities · Large deviations · Reproducing Kernel Hilbert Space (RKHS) · Small deviations

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