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Thomas H. Otway

# The Dirichlet Problem for Elliptic-Hyperbolic Equations of Keldysh Type

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Thomas H. Otway
Department of Mathematical Sciences
Yeshiva University
New York
USA

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# Preface

Partial differential equations of mixed elliptic–hyperbolic type arise in diverse areas of physics and geometry, including fluid and plasma dynamics, optics, cosmology, traffic engineering, projective geometry, geometric variational theory, and the theory of isometric embeddings. And yet even the linear theory of these equations is at a very early stage. This course examines various Dirichlet problems which can be formulated for equations of Keldysh type, one of the two main classes of linear elliptic–hyperbolic equations. Open boundary conditions (in which data are prescribed on only part of the boundary) and closed boundary conditions (in which data are prescribed on the entire boundary) are both considered. Emphasis is on the formulation of boundary conditions for which solutions can be shown to exist in an appropriate function space. Specific applications to plasma physics, optics, and analysis on projective spaces are discussed.

These notes were written to supplement a series of ten lectures given at Henan University in the summer of 2010. They are intended for graduate students and researchers in pure or applied analysis. In particular, the reader is expected to have a background in functional analysis – including Sobolev spaces – and the basic theory of partial differential equations, but not necessarily any prior expertise in the theory of mixed elliptic–hyperbolic equations. A familiarity with the geometry of differential forms is assumed in Sect. 5.6, but that material can be skipped without loss of continuity. Although some mathematical ideas which are used frequently in the text are collected in Chap. 2, that material is not intended to replace the above-listed prerequisites.

As is typical with monographs of this kind, some of the results have not previously appeared in the literature. Examples are Theorems 3.1–3.3, 6.2, and 6.3. But the proofs of those results are based on rather standard arguments for this field. We also include examples of results which have been asserted in the literature but for which detailed proofs are missing or obscure; an example of such "folklore" is Theorem 4.1. In all cases the form of the included results has been dictated more by expository considerations than by a desire to extend the research literature.

There has been considerable research into elliptic–hyperbolic equations in China, Russia, and Bulgaria. Unfortunately, some of the older literature of those countries remains difficult to access, and there are undoubtedly important contributions which I have missed. I would be grateful to the readers for comments on lacunae in the references and, of course, on errors or omissions in the text.

I am grateful to Daniela Lupo and Kevin R. Payne for discussion of Proposition 4.1, and to Antonella Marini for discussion related to Theorem 6.3. I am also grateful to Yisong Yang for suggesting the course on which these notes are based (and for many conversations about mathematical physics and analysis), to Yuxi Zheng for independently suggesting that I write a set of lecture notes on the subject (and also for many conversations about mathematical physics and analysis), and to Ke Wu for inviting me to give the lectures at Henan University. These notes have benefited substantially from the comments of anonymous referees for *Lecture Notes in Mathematics*. It has been a pleasure to work with the excellent editorial/production staff assigned to this series. Finally, I thank Shouxin Chen and the administration, faculty, and graduate students of the School of Mathematics and Information Science at Henan University for their hospitality during the week in which the lectures were given, and my fellow lecturer Joel Spruck for his collegiality during that time.

New York City *Thomas H. Otway*

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