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Ulrich Ellwanger

# From the Universe to the Elementary Particles

A First Introduction to Cosmology and  
the Fundamental Interactions

 Springer

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*To my wife Gabi*

# Preface

It is remarkable that the fundamental laws of nature are simple. The complexity of the processes in our environment can be traced back to the fact that matter—gases, liquids, and solids—consists of an enormous number of building blocks (atoms and molecules).

Only in exceptional cases do the processes in our environment reflect the simplicity of the laws of nature. The relatively simple law of gravity allows for a description of the motion of planets or the free fall of a heavy body—but only if friction can be neglected. Even the description of the trajectory of a falling sheet of paper, where friction and other forces are important, becomes extremely complicated.

Moreover, the fundamental laws of nature seem to become progressively simpler the deeper one penetrates into the world of elementary building blocks from atoms to elementary particles. For instance, the numerous electric and magnetic phenomena can be traced back to a simple theory of electromagnetism.

However, the simplicity of such a theory reveals itself only if one employs mathematical formulations corresponding to those nature seems to use. This fact is remarkable by itself. Consequently, a certain mathematical equipment is required in order to understand the laws of nature. During recent decades this understanding has made enormous progress. We understand most of the processes in particle physics and cosmology, and manage to describe them in simple terms after making use of appropriate mathematical concepts.

The aim of this book is to present the current status of our knowledge of the laws of nature from cosmology to the elementary particles. However, we also address the numerous open questions, which often relate, interestingly enough, phenomena in cosmology to phenomena in particle physics.

The current status of our knowledge of the laws of nature encompasses four fundamental forces—gravity, electromagnetism, and the strong and the weak interactions—as well as a few elementary particles as “building blocks.” Possible answers to open questions are theories that, up to now, could be neither confirmed nor disproved by experiments: amongst others, theories of the unification of three of the four fundamental forces (the exception being gravity), supersymmetry, and

string theory. These theories will also be described briefly. Particle physicists and cosmologists hope to learn from experimental results within the next few years whether (or which) one of the presently speculative theories does actually describe nature.

In this text we presuppose the mathematical level of knowledge of students in natural sciences at the beginning of their studies: vector calculus, derivatives, simple differential equations and integrals. In addition, it is necessary to introduce several concepts that play an important role in cosmology and particle physics: special and general relativity, as well as classical and quantum field theory. Obviously it is impossible to describe these concepts in all detail, which would require considerably more complicated mathematical formalisms as well as a complete series of books. However, we present the essential aspects of these concepts, and many phenomena can be understood with the help of calculations that are feasible using the above mathematical equipment. In this respect, the text goes beyond the level of popular science.

The text starts with a review, beginning with the largest possible structure—the Universe—and passing by atoms and nuclei to the elementary particles, the quarks and leptons. Subsequently the corresponding concepts and physical phenomena are discussed in detail. At the end we briefly sketch the currently still speculative theories mentioned above. The text should allow a scientifically interested reader to share the fascination that accompanies penetration into the fundamental laws of nature, and possibly into a theory unifying all fundamental forces.

Finally, I would like to take this opportunity to thank several readers and Prof. D. Gromes for suggestions that helped to improve this book.

Orsay, September 2011

Ulrich Ellwanger

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