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Klaus Mainzer · Leon Chua

The Universe as Automaton

From Simplicity and Symmetry
to Complexity

 Springer

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Preface

This booklet is an essay at the interface of philosophy and complexity research, trying to inspire the reader with new ideas and new conceptual developments in cellular automata (CA). Although the text is introductory, it goes beyond the presentation of nice pictures with pattern formations. Steven Wolfram declared computer experiments with pattern formation by CA as a “new kind of science”. We claim that even in the future, quasi-empirical computer experiments are not sufficient. CA must be considered complex dynamical systems in the strictly mathematical sense, with corresponding equations and proofs. In short, we need analytical models for CA, to find precise answers and predictions in the universe of CA. In this sense, our booklet goes beyond Wolfram’s approach.

After a historical and philosophical introduction to the old question “Is the universe a (cellular) automaton?” CA are defined as complex dynamical systems. The geometrical representation of the eight CA-rules as a Boolean cube allows precise definitions of a complexity index and universal symmetries. It can be proved that the 256 one-dimensional CA are classified by local and global symmetry classes for CA. There is an exceptional symmetry group with universal computability which we call the “holy grail” in the universe of CA. Although the four automata of this group are completely deterministic, their long-term behavior cannot be predicted in principle with respect to the undecidability of Turing’s famous halting problem. Many analytical concepts of complexity research (such as attractors, basins of attractors, time series, power spectra, and fractality) are defined for CA. But there are also surprising phenomena in the CA-world (isles of Eden) without analytical representation in dynamical systems.

Finally we ask whether CA can be considered models of the real world? We introduce a test procedure to decide between an arrow of time or time reversibility in the attractor dynamics of CA. Can we compare symmetries of the physical universe with symmetries in the toy world of CA? What are the similarities and differences? According to a famous hint by Richard Feynman, classical probabilistic and nondeterministic automata are not sufficient to simulate the quantum universe. Therefore, quantum CA are a promising field for future research. CA can also be considered models of complex networks in the life sciences and

technology. We discuss applications in systems biology, brain research, and robotics. Self-organization and the emergence of structure and patterns can be made precise in the CA-context.

In the end it is not essential whether the universe is an automaton in some metaphysical sense. In any case, CA are beautiful and fascinating examples of a general tendency in modern research: The world is increasingly represented by digitized models to handle the increasing complexity of research by high-speed computers. The final philosophical question arises of whether the digitization of the world has limitations.

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Munich and Berkeley, April 2011

Klaus Mainzer and Leon Chua

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