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# Correction Formulae for the Stress Distribution in Round Tensile Specimens at Neck Presence

Springer

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ISSN 2191-5342
e-ISSN 2191-5350
ISBN 978-3-642-22133-0
e-ISBN 978-3-642-22134-7
DOI 10.1007/978-3-642-22134-7
Springer Heidelberg Dordrecht London New York



*Cover design:* eStudio Calamar, Berlin/Figueres

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

# Preface

This monograph is devoted to one of the most popular methods for the determination of the plastic material properties, i.e. the tensile test particularly from the moment of neck appearance in the sample.

Despite the fact that a few different classical formulae exist which describe the stress distribution in the neck, there is not any certainty which of them is more accurate and the choice to apply a certain formula is often somewhat arbitrary. After thorough literature search, it turned out that the formula of Bridgman is more often utilised in practice for the yield stress determination. However, our preliminary analysis has shown that it may generate rather non-negligible error (up to ten percent) at least in the case of ideal plastic materials.

It is well known that in the western literature Bridgman's formula is more frequently used while the eastern literature prefers the approximation by Davidenkov-Spiridonova. Both of these formulae were derived in the forties of the last century. What is interesting is that for the first time a formula for the determination of the average normalised axial stress in the minimal section plane was derived by Siebel, which indeed overlaps with the approach proposed by Davidenkov-Spiridonova. Siebel's work is however less often used and its relative obscurity can probably be historically explained by the fact that it was published in Germany shortly after the Second World War. Obviously, repeated trials were made to derive more accurate formulae and at least two of them were successful (Szczepiński's, Malinin's) but the obtained solutions are still seldom utilised in practice also because of a lack of information on their accuracy in comparison with the classical formulae.

The authors' aim in the presented monograph is to collect all known results in the area and to answer the aforementioned questions. Indeed, one can find in the detailed description of materials flow curves determination, criteria of neck creation, derivations of all known formulae for stress distribution in the neck of axisymmetric samples as well as estimation of accuracy of simplifying assumptions applied during the derivation of the classical formulae by means of very accurate numerical simulations. As a result of the critical analysis of the simplifications, a new empirical formula was derived which depends on the same geometrical parameter (i.e. ratio of the sample radius in the minimal section to the contour radius of the deformed sample) as the classical formulae, but revealing higher accuracy than them. In addition, a new analytical model was proposed, which describes the stress distribution in the neck of an axisymmetric tensile specimen and on its basis a new formula for the average normalised axial stress in the minimum section plane was derived. This formula takes into account in addition to the mentioned parameters a new ratio (i.e. the relative neck radius in a measure of its deformation). Fortunately, both aforementioned parameters can be easily measured in experimental tests. During the verification of all formulae based on data obtained from the numerical simulation, it turned out that this new formula reveals higher accuracy in comparison with residuals.

This monograph is recommended for students and PhD students enrolled in mechanics and materials technology courses, scientists interested in experimental mechanics and engineers dealing with the determination of elasto-plastic material properties from experiments.

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# Contents

# Notation

| Variable | Explanation |
|---|---|
| $D_\varepsilon = \varepsilon - \varepsilon^0$ | Deviator of strain tensor |
| $D_\sigma = \sigma - \sigma^0$ | Deviator of stress tensor |
| $F$ | Tensile force |
| $I_1(\sigma) = \mathrm{Tr}\sigma = \sigma_r + \sigma_z + \sigma_\theta$ | First tensor invariant |
| $I_2(\sigma) = \frac{1}{2}[\mathrm{Tr}\sigma^2 - \mathrm{Tr}^2\sigma]$ | Second tensor invariant |
| $I_3(\sigma) = \det\sigma$ | Third tensor invariant |
| $J_1(\varepsilon) = 0$ | First deviator invariant |
| $J_2(\varepsilon) = I_2(\varepsilon) + \frac{1}{3}I_1^2(\varepsilon)$ | Second deviator invariant |
| $J_3(\varepsilon) = I_3(\varepsilon) + \frac{1}{3}I_1(\varepsilon)I_2(\varepsilon) + \frac{2}{27}I_1^3(\varepsilon)$ | Third deviator invariant |
| $J_0(\xi)$ | Bessel function of zero order |
| $J_1(\xi)$ | Bessel function of first order |
| $K$ | Elastic bulk modulus |
| $L_0$ | Initial length of sample |
| $R$ | Curvature radius of contour of the deformed sample |
| $R_{\mathrm{eH}}$ | Upper yield point |
| $R_{\mathrm{eL}}$ | Lower yield point |
| $R_{\mathrm{m}}$ | Ultimate strength |
| $R_{\mathrm{p0.2}}$ | Conventional yield point |
| $S$ | Current cross section area of sample |
| $S_0$ | Initial cross section area of sample |
| $V$ | Volume of material |
| $a$ | Current sample radius in the minimum cross section |
| $a_0$ | Initial radius of sample |
| $k$ | Yield stress |
| $\bar{k}$ | Average yield stress |
| $\bar{\mathbf{u}}$ | Displacement vector |
| $r$ | Radial coordinate in current configuration |
| $r = a_z(z)$ | Radius function of the neck cross section at a distance $z$ from the plane of the minimal section |
| $\bar{\mathbf{v}}$ | Velocity vector |

(continued)

(continued)

| Variable | Explanation |
|---|---|
| $z$ | Axial coordinate in current configuration |
| $\Delta L$ | Sample elongation |
| $\Delta\varepsilon_{\text{int}} = (\varepsilon_{\text{int}} - \bar{\varepsilon}_{\text{int}})/\bar{\varepsilon}_{\text{int}}$ | Relative increase of the strain intensity |
| $\Lambda = 1 - a_0/a$ | Parameter characterising the stage of plastic strain accumulated in the whole sample |
| $\Phi(\sigma_{ij})$ | Plastic potential |
| $\Psi = \Psi(J_2(\varepsilon))$ | Known function in the deformation theory of plasticity linking the strain and stress deviators |
| $\alpha, \beta$ | Parameters describing function of the curvature radius of the longitudinal stress trajectory |
| $\delta = a/R$ | Parameter describing the stage of strain location in the neck surrounding |
| $\varepsilon = \Delta L/L_0$ | Engineering strain (Cauchy's strain) |
| $\bar{\varepsilon} = \ln\frac{L}{L_0} = \ln(1+\varepsilon)$ | Logarithmic strain (Hencky's strain) |
| $\underline{\boldsymbol{\varepsilon}}$ | Strain tensor |
| $\dot{\underline{\boldsymbol{\varepsilon}}}$ | Strain rate tensor |
| $\bar{\varepsilon}^{el}$ | Logarithmic elastic strain |
| $\varepsilon^0 = \frac{1}{3}I_1(\varepsilon)$ | Isotropic component of stain tensor |
| $\varepsilon_{\text{int}}$ | Strain intensity (defined by means of the second invariant of the strain deviator) |
| $\bar{\varepsilon}_{\text{int}}$ | Average strain intensity in the minimal section plane |
| $\bar{\varepsilon}^{\text{p}} = \ln(1+\varepsilon) - (1-2v)\frac{\sigma_{\text{true}}}{E}$ | True (logarithmic) plastic strain up to the moment of neck creation |
| $\bar{\varepsilon}^{\text{pl}} = 2\ln\left(\frac{a_0}{a}\right)$ | True (logarithmic) plastic strain from the moment of neck creation |
| $\theta$ | Circumferential coordinate in current configuration |
| $\lambda$ | Factor of proportionality in the constitutive equation of the plastic flow theory determined from the yield condition for each stage of deformation |
| $\lambda$ | Lamé's elasticity coefficient |
| $\mu$ | Lamé's elasticity coefficient |
| $v$ | Poisson's ratio |
| $\rho$ | Curvature radius of the principal stress trajectory $\sigma_3$ |
| $\underline{\boldsymbol{\sigma}}$ | Stress tensor |
| $\sigma^0 = \frac{1}{3}I_1(\sigma)$ | Isotropic component of stress tensor |
| $\sigma_0 = F/S_0$ | Engineering stress |
| $\sigma_{\text{true}} = F/S$ | True stress (responding the average axial stress in the minimal section plane $\bar{\sigma}_z$); as distinguished from the shearing stress - $\tau_{\text{rz}}$ |
| $\psi$ | Slope angle of the tangent of principal stress trajectory $\sigma_3$ to the axis $z$ |