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Asynchronous Operators of Sequential Logic: Venjunction & Sequention

Digital Circuit Analysis and Design

Springer

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Preface

"What is to be done?"
Nikolay Chernyshevsky

First of all let me comply with A.L. Fradkov[1]: "Overall majority of science areas today look like a flowering meadow after a huge herd of animals had a walk on it. Grass is largely eaten and "juiciest peaces" are eroded up to the ground. Some areas are strolled upon numerous times. Here and there grass is only rumpled, but it would take an enormous effort to eat it, and the result would be so hardly noticeable that big animals are already seeking new pastures over many years…"

Now in essence.

This manuscript is dedicated to new mathematical instruments assigned for logical modeling of the memory of digital devices. The case in point is logic-dynamical operation named venjunction and venjunctive function as well as sequention[2] and sequentional function. Venjunction and sequention operate within the framework of sequential logic. In a form of corresponding equations, they organically fit analytical expressions of Boolean algebra. Thus, a sort of symbiosis is formed using elements of asynchronous sequential logic on the one hand, and combinational logic on the other hand. Thence, a common denomination is asynchronous logic.

A peak of publications on the asynchronous logic came in the middle of the 1990s. In spite of undertaken efforts and progress observed in this direction, it must be confessed that properly appreciable results are not achieved. At least not such, as have been expected. Together with this, relevance and claiming of asynchronous circuit as an object of science interest does not raise doubts. But it is evident that priority of investigations is displaced to programming and verification aspects as well as to the practical field of technology and technical engineering.

A wealth of methods, ways and applications developed for solving various problems related with digital circuits allow establishing the following.

- Asynchronous circuit analysis and design is a complicated and heterogeneous problem.
- There are no universal generally accepted approaches.

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[1] Article "How to publish a good article and to reject a bad one. Notes of a reviewer" Available in Russian at: http://www.ipme.ru/ipme/labs/ccs/alf/f_at03r.pdf.

[2] Not to be confused with "sequent", used for the sequent calculus.

- All models and methods, as well as their applications, are not devoid of disadvantages.
- New ideas are desirable and required.

In the light of established situation, a curious thought arises: may be finite-automaton apparatus as being monopoly mathematical instrument for sequential circuits is the cause of intractable problems, because its scientific resource is exhausted.

Indeed?!

Foreword

In the beginning was Boolean algebra. Named in honor of George Boole, this algebra gave symbolic expressions for a binary logic. So the beginning relates to the 1840s.

Development of the theoretical grounds for Boolean algebra's digital applications has been basically completed in the 1950s. At that time mathematical instruments for representation, transformation and minimization of logical operations were ready

Boolean algebra constitutes a mathematical basis for digital circuits. It is known, how relatively easy Boolean equation is translated into logical circuit. Inverse transformation is performed without difficulties as well. In other words, a principle of one-to-one correspondence is on hand. Boolean expressions play the role of a natural analytical model of combinational digital circuits.

In contrast with combinational logic, there are no easy ways to represent behavior of logical circuits with feedbacks in an analytical form; tables and graphs are in use. Circuit's logic language is not in accord with state-transition expressions. Therefore, constructing of sequential circuit is a complicated, many-staged and heterogeneous procedure. As to inverse transformation, present analysis does not give formal methods for this problem. The corresponding solution is available only for simple circuits with memory elements.

The problem is how to formalize operation of memory devices mathematically in terms of binary logic, and how to get analytical model which enables to adequately reflect functionality of these devices. In this connection it is proposed to solve the mentioned problem by means of asynchronous operators, named venjunction and sequention. Under conditions that existing delays are unknown, algebraic expressions obtained on the basis of these operations one-to-one correspond to the modeled circuit. Thus, relation between asynchronous sequential circuit and its model becomes similar to relation between combinational circuit and expressions of Boolean algebra. As a result new analytical possibilities appear and some problems could be solved in easier way. Asynchronous operators may be found to be not only useful, but productive as well.

The book contains initial concepts, fundamental definitions, statements, principles and rules needed for theoretical justification of the new mathematical apparatus and its validity for asynchronous logic. Asynchronous operators named venjunctor and sequentor are designed for practical implementation. These basic elements are assigned for realizing of memory function in sequential circuits. It is important that in general case memory depth theoretically is not restricted.

The author develops original approach, whose superiority is in homogeneity of mathematical expressions. This approach does not substitute or copy the existing procedures; as well it does not solve all problems. Obtained results can be used independently or together with present methods.

Formed sequential logic essentially is classified as switching logic, which falls into category of algebraic logics. For better understanding reader should know Boolean algebra, set theory, and foundations of digital devices.

Present research work is the final stage of generalization and systematization of all those ideas and investigations, author's interest to which alternately flashed up and faded over many years and for various reasons until formed "critical mass", and all findings were arranged definitively as a mathematical basis of a theory appropriately associated under a common theme – asynchronous sequential logic.

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