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Florian Scheck

Electroweak and Strong Interactions

Phenomenology, Concepts, Models

Third Edition

With 59 Figures

Springer

Prof. Dr. Florian Scheck
Universität Mainz
Institut für Physik Theoretische
Elementarteilchenphysik
Staudinger Weg 7
55099 Mainz
Germany
scheck@uni-mainz.de

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To the memory of Res Jost, who was an outstanding scientist and a truly exceptional personality

Aber sage nur niemand, daß uns das Schicksal trenne! Wir sinds, wir! Wir haben unsere Lust daran, uns in die Nacht des Unbekannten, in die kalte Fremde irgend einer andern Welt zu stürzen, und wär' es möglich, wir verließen der Sonne Gebiet und stürmten über des Irrsterns Grenzen hinaus.

(Friedrich Hölderlin, Hyperion, 1. Band, 1. Buch)

But never let it be said that Fate puts us asunder. We do, we ourselves, we delight in flinging ourselves into the dark unknown, into the cold abroad of some other world, we'd leave the zone of the sun altogether if we could and career beyond the frontiers of the wandering star.

(Hyperion by F. Hölderlin, Vol. 1, Part 1, 1797) translation taken from D. Constantine, "Hölderlin", Clarendon Press, Oxford, 1988, p. 348

Preface to the Third Edition

This book has its roots in a book on *Leptons, Hadrons and Nuclei* which I published, under that title in 1983, and, naturally enough, in the lectures and courses on elementary particle physics that I have given over the years, first at the Eidgenössische Technische Hochschule in Zurich, and later at the Johannes Gutenberg University in Mainz. Since 1983 – the year of the discovery of the W and Z bosons – experimental tests of what is now called the standard model of electroweak and strong interactions have made dramatic progress and, in fact, have reached a qualitatively new level. It now appears that the standard model, in its minimal version, is very well confirmed and, as yet, there is little hint of physics at higher mass scales, going beyond the standard model. This consolidation of the standard model concerns not only its building blocks and its general pattern but also the radiative corrections it predicts. For the student and for the beginner in the field of particle physics it is important to learn about the way to the standard model, the experimental basis on which it rests, its predictive power and its limitations. In particular, the novice in theoretical particle physics who sets out either to find a better foundation for the model, or else to recklessly dethrone it, hence to revolutionize our field, should know where he or she is sailing and what he or she is searching for. This is the reason why I decided to concentrate on the foundations and the phenomenology of electroweak and strong interactions rather than giving yet another account of the intricacies of quantized gauge theory.

There are many excellent textbooks and monographs on quantized field theory (for example, [ITZ80], [ZIJ94], [COL84], [DWS86]) and, more specifically, on quantized gauge field theory ([CHL84], [HUA92], [OKU82], [BEB94] and many more), but only few books covering in depth the phenomenology or the contact to nuclear physics (noteworthy exceptions are [PER87], [NAC94], [POR95]). This book is at the level of what might be termed *advanced quantum mechanics;* that is, I assume that the reader is familiar with nonrelativistic quantum mechanics and with the foundations of special relativity. In writing it I made every effort to define, to explain and to illustrate the basic notions and to explicitly show the path from them to the final physical results. Although the level is not elementary, with a little effort and perseverance, the reader, whether experimentally or theoretically oriented, should be able to follow the complete argument or derivation in every subject that this book addresses, without having to resort to other sources. I do hope, of course, that this aspect will contribute to the fun and the satisfaction in learning the topics dealt with in this book. Sections marked with an asterisk contain more detailed material that may be skipped in a first reading.

Being largely self-contained, the book can be read as an independent text by anyone eager to learn this physics or to refresh his or her knowledge. As it originated in graduate-level lectures it may also serve as an accompanying textbook for a one- or two-semester course, perhaps with some cuts. Every chapter is followed by a set of exercises, some of which are simple whereas others require a little more time and effort. Solutions to selected exercises are given at the end of the book. All exercises will help the reader to test his or her understanding, and some serve the purpose of further illustrating the content of the corresponding chapter.

As compared to the second edition of 1996 various new developments and experimental results in electroweak physics are brought to date. The sections on deep inelastic scattering and QCD in Chaps. 2 and 3, as well as the discussion of neutrino oscillations in Chap. 4 are revised and extended. In turn, the fields of hadron scattering on nuclei and of hadronic atoms no longer are in focus of present-day experimental and theoretical research. Therefore the two chapters dealing with these topics were dropped here. If the need arises they may be consulted in the earlier edition of 1996.

In a field as vast and rapidly expanding as particle physics, the bibliography is bound to be incomplete, biased and to some extent unbalanced. I have adopted the following compromise: Within each chapter and at its end I give a selection of references, mostly on experimental results, which have direct bearing on the content of the chapter. In addition, towards the end of the book, there is a list of handbooks, textbooks and monographs to which I refer throughout all chapters using the notation [ABCxy](author(s) and year). Anyone who wishes to delve deeper into some topic dealt with in this book is advised to turn first to the review articles quoted here which will be helpful in retracing the complete literature.

Theoretical physics is a synthesis of lonely work and lively interaction with others. My colleagues and friends, my collaborators and students from whom I learnt a great deal and who directly or indirectly contributed to the genesis of this book, are too numerous to list here. I am very grateful to all of them for much stimulation, fruitful criticism, lively discussions or simply for the pleasure of collaborating with them.

Let me, quite presumptiously, adapt for my purpose the beautiful dedication that Johann Sebastian Bach chose for his Well-Tempered Clavier in 1722,

> “Zum Nutzen und Gebrauch der Lehr-begierigen *Physicali*schen Jugend, als auch derer in diesem *studio* schon *habil* seyenden besonderem Zeit Vertreib auffgesetzet und verfertiget…”

which means "Written and composed both for the benefit and use of young physicists desirous of instruction and for the particular diversion of those already advanced in this study...", (translation taken from British Library Music Facsimiles I, The British Library, 1980).

Mainz *Florian Scheck*
August 2011

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Notation and Conventions

To a large extent, the notation is explained in the text. Nevertheless it may be useful to first look through this short section on notations and conventions, or to return to it when one is not absolutely sure about a symbol or definition that is used in the text.

(i) *Units*. We use natural units

$$\hbar = 1, c = 1$$

throughout this book. Taking $c = 1$ means that coordinates and time have the same dimension, $[q_{\text{nat}}] = [t_{\text{nat}}]$, likewise, momenta and energy have the same dimension $[p_{\text{nat}}] = [E_{\text{nat}}]$. As in a physical system of units (SI, Gauss, or other) $\hbar$ has dimension energy $\times$ time and as this is also the dimension of an *action* such as $p \cdot q$, taking this constant equal to 1 means that

$$[p_{\text{nat}}] = [E_{\text{nat}}] = [q_{\text{nat}}]^{-1} = [t_{\text{nat}}]^{-1}.$$

This convention is not sufficient to fix the units completely. What is needed, in addition, is a unit of energy (or mass, or time). Following standard practice, we use multiples of the electron Volt,

$$1 \text{ meV} = 10^{-3}\,\text{eV} \quad 1 \text{ keV} = 10^{3}\text{ eV}$$

$$1 \text{ MeV} = 10^{6}\,\text{e}V \quad 1 \text{ GeV} = 10^{9} \text{ eV} \quad 1 \text{ TeV} = 10^{12}\,\text{eV},$$

i.e., milli, kilo, mega, giga, tera eV, respectively.

It is easy to translate length l, cross section σ, and time t from natural units back to conventional units. Let $\mathring{l}$, $\mathring{\sigma}$, $\mathring{t}$ be such quantities expressed in natural units, l, σ, t the same quantities in standard units. Then

$$l\,[\mathrm{fm}] \stackrel{\Delta}{=} \hbar c \cdot \mathring{l}\,[\mathrm{MeV}^{-1}],$$

$$t\,[\mathrm{s}] \stackrel{\Delta}{=} \frac{\hbar c}{c} \cdot \mathring{t}\,[\mathrm{MeV}^{-1}],$$

$$\sigma\,[\mathrm{fm}^2] \stackrel{\Delta}{=} (\hbar c)^2 \cdot \mathring{\sigma}\,[\mathrm{MeV}^{-2}],$$

with $\hbar c = 197.3270\,\mathrm{MeV} \cdot \mathrm{fm}$, $1\ \mathrm{fm} = 10^{-13}\ \mathrm{cm}$, $c = 2.99792458 \times 10^{10}\ \mathrm{cm/s} = 2.99792458 \times 10^{23}\ \mathrm{fm/s}$. Cross sections are usually expressed in units of $1\ \mathrm{b} = 10^{-24}\ \mathrm{cm}^2 = 10^2\ \mathrm{fm}^2$. Thus, if $\mathring{\sigma}$ is found in GeV^{-2}, for example, then it follows that

$$1\ \mathrm{GeV}^{-2} \stackrel{\Delta}{=} 0.3894\,\mathrm{mb}.$$

Another example is the relationship between the width Γ and the lifetime τ of an unstable state,

$$\Gamma = \left(\frac{\hbar c}{c}\right)\frac{1}{\tau} = 6.58212 \times 10^{-22}\mathrm{MeVs}\frac{1}{\tau}.$$

Momenta p are expressed in energy units if h and c are set equal to one, and are given in (energy unit)/c when conventional units are used.

(ii) *Experimental results and errors* are generally quoted with the error of the last digits in parentheses. For example,

$$0.7773(13) \text{ means } 0.7773 \pm 0.0013,$$

$$0.51126(5) \text{ means } 0.51126 \pm 0.00005.$$

The abbreviation ppm stands for "parts per million". For example, a measurement giving the result 0.510 999 06(15) MeV for the electron mass is a "0.3 ppm measurement".

(iii) *Metric and normalization*. The metric is explained in more detail in Chap. 1. We use the form $g^{00} = +1$, $g^{ii} = -1$ ($i = 1, 2, 3$) for the diagonal metric tensor. A contravariant vector is denoted by $x^\mu = (x^0, \boldsymbol{x})$ so that $x_\mu = (x^0, -\boldsymbol{x})$. One-particle states appear with the covariant normalization

$$< \boldsymbol{p}'|\boldsymbol{p} >= 2E_p\delta(\boldsymbol{p} - \boldsymbol{p}')$$

for both bosons and fermions. In the case of particles with spin there is an additional Kronecker δ-symbol for the spin indices.

(iv) *Some symbols. T* denotes the scattering matrix and is defined in App. B.

$\overleftrightarrow{\nabla}$ and $\overleftrightarrow{\partial}_\mu$ are short-hand notations for antisymmetric derivatives

$$f(\boldsymbol{x})\overleftrightarrow{\nabla}g(\boldsymbol{x}) = f(\boldsymbol{x})(\boldsymbol{\nabla}g(\boldsymbol{x})) - (\boldsymbol{\nabla}f(\boldsymbol{x}))g(x),$$

$$f(x)\overleftrightarrow{\partial}_\mu g(x) = f(x)(\partial_\mu g(x)) - (\partial_\mu f(x))g(x).$$

A list of symbols is found below.
(v) *Rotation matrices*. For the representation coefficients of the rotation group we use the definitions of e.g. [FAR59], that is

$$D^{(j)}_{KM}(\psi,\theta,\phi) = \mathrm{e}^{iK\psi} d^{(j)}_{KM}(\theta)\mathrm{e}^{iM\phi}$$

with Euler angles as defined in Fig. 1.1 (p.12). As explained in more detail in Sect. 1.2, these are the matrices which transform the expansion coefficients. Basis functions then transform according to D^*.

(vi) *Natural units for Maxwell's equations*. With $\boldsymbol{E}$ and $\boldsymbol{H}$ denoting the electric and magnetic fields, respectively, and $\boldsymbol{D}$ and $\boldsymbol{B}$ the electric displacement and magnetic induction, respectively, Maxwell's equations read in any system of units

$$\nabla\cdot\boldsymbol{B} = 0, \quad \nabla\times\boldsymbol{E} + f_1\tfrac{\partial\boldsymbol{B}}{\partial t} = 0$$
$$\nabla\cdot\boldsymbol{D} = f_2\rho, \quad \nabla\times\boldsymbol{H} - f_3\tfrac{\partial\boldsymbol{D}}{\partial t} = f_4\boldsymbol{j}\,.$$

With q denoting the electric charge, the Lorentz force is

$$\boldsymbol{F} = q(\boldsymbol{E} + f_1\boldsymbol{v}\times\boldsymbol{B}).$$

As is well known from electrodynamics, the fundamental fields are $\boldsymbol{E}$ and $\boldsymbol{B}$. In vacuum the derived fields $\boldsymbol{H}$ and $\boldsymbol{D}$ are related to the former by

$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E}, \quad \boldsymbol{B} = \mu_0\boldsymbol{H}.$$

By convention the constants ε_0 and μ_0 are chosen in such a way that $f_1 = f_3$. Furthermore, the continuity equation which follows from the two inhomogeneous Maxwell equations, requires the relation $f_4 = f_1 f_2$, thus leaving three constants to be fixed by a suitable choice of physical units: f_1, f_2, and ε_0. The reader will easily verify that the SI system is based on the choice $f_1 = f_2 = 1$ and $\varepsilon_0 = 10^7/(4\pi c^2)$, while the Gauss system is characterized by the choice $f_1 = 1/c, f_2 = 4\pi$, $\varepsilon_0 = 1$.

The *system of natural units* that is used in elementary particle physics takes the velocity of light c to be unity, (as well as Planck's constant divided by 2π, $\hbar = 1$), cf. (i) above, and is also designed such that the factors 4π disappear from Maxwell's equations. This means setting

$$f_1 = f_2 = 1, \quad \varepsilon_0 = \mu_0 = 1$$

and absorbing square roots of 4π into the fields and charges as follows,

$$E|_{\mathrm{nat}} = \frac{1}{\sqrt{4\pi}}E|_{\mathrm{Gauss}},\; B|_{\mathrm{nat}} = \frac{1}{\sqrt{4\pi}}B|_{\mathrm{Gauss}},$$

$$\rho|_{\mathrm{nat}} = \sqrt{4\pi}\,\rho|_{\mathrm{Gauss}}, \quad j|_{\mathrm{nat}} = \sqrt{4\pi}\,j|_{\mathrm{Gauss}},$$

Obviously, this is a convenient framework for doing calculations. When returning to customary units such as Gauss' system, and evaluating the results of a calculation, all that remains to be done at the end is to multiply external fields by $\sqrt{4\pi}$ and to replace the squared elementary charge e^2 by $4\pi\alpha$, where α is Sommerfeld's fine-structure constant. Indeed, from what we said above, we have

$$\alpha = \frac{e^2|_{\text{Gauss}}}{\hbar c} = \frac{e^2|_{\text{nat}}}{4\pi} \approx \frac{1}{137.036}$$

(vii) All equations, figures, tables and sections are numbered beginning with the number of the chapter in which they occur. They can thus easily be located when they are mentioned elsewhere.

List of Symbols

$A_\alpha(x) = \mathrm{i}e \sum_k^N T_k A_\alpha^{(k)}(x)$:	gauge potential taking its values in the Lie algebra spanned by generators T_k
$a_\alpha(x)$:	weak axial vector current
a_B:	Bohr radius of atomic orbit
$\boldsymbol{C}$:	charge conjugation operator
$\mathbb{C}$:	field of complex numbers
$D_\alpha(A) = \mathbb{1}\partial_\alpha + A_\alpha$:	covariant derivative
∂_μ:	partial derivative with respect to x^μ, with x a point in Minkowski space-time
∂^μ:	partial derivative with respect to x_μ, with x a point in Minkowski space-time
ε_{ijk}:	totally antisymmetric tensor in 3 real dimensions
$\varepsilon_{\mu\nu\sigma\tau}$:	totally antisymmetric tensor in 4 real dimensions (convention $\varepsilon_{0123} = +1$)
$f(x) \equiv \psi^{(f)}(x)$:	shorthand for Dirac field describing fermion f
$F_{\alpha\beta}(x) = \mathrm{i}e \sum_k^N T_k F_{\alpha\beta}^{(k)}(x)$:	field strength tensor of Abelian ($N = 1$) or non-Abelian gauge theory
$f_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$:	kinetic part of field strength tensor $F_{\alpha\beta}$
${}_1F_1\ (a; b; x)$:	confluent hypergeometric function
$F_i(q^2)$:	form factors in hadronic matrix elements of electromagnetic and weak vector currents
$F_\mathrm{A}(q^2),\ F_\mathrm{P}(q^2)$:	axial and pseudoscalar form factors for weak axial current
f_π:	decay constant of charged pion
$\phi_a(x),\ a = 1, 2$:	spinor field of first kind
$\chi^A(x),\ A = \mathrm{I, II}$:	spinor field of second kind
$G/\sqrt{2} = g^2/(8m_\mathrm{w}^2)$:	Fermi's constant (G) of weak interactions

g, g' or e:	(dimensionless) coupling constant in gauge theories
$g_{\mu\nu}$, $g^{\mu\nu}$:	metric tensor in Minkowski space–time
$G_{\mathrm{E}}(q^2)$, $G_{\mathrm{M}}(q^2)$:	electric and magnetic form factors of nucleon
$\mathscr{H}$:	Hamiltonian density
$h(\nu_f)$:	helicity of neutrino in lepton family $f = e, \mu$ or τ
$\mathbb{1}$:	unit matrix or, more generally, identity operation
$\mathscr{L}$:	lagrangian density
L_f:	lepton family number for $f = e, \mu$ or τ
Λ:	Lorentz transformation
$L(\boldsymbol{v})$:	special Lorentz transformation or boost
$\mathring{M}$:	diagonal form of a hermitean or symmetric matrix
$\boldsymbol{P}$:	space-reflection or parity operation
ψ_L, ψ_R:	left- and right-handed components of Dirac field
$\overline{\psi(x)} = \psi^\dagger(x)\gamma_0$:	conjugate Dirac field
$\mathscr{R}$:	rotation in four dimensions, $\mathscr{R} = \mathrm{diag}(1, \boldsymbol{R})$, where $\boldsymbol{R} \in \mathrm{SO}(3)$ is a rotation matrix
$\mathbb{R}$:	field of real numbers
$S = \mathbb{1} + R$:	S-matrix, decomposed into the identity (“no scattering”) and the reaction matrix R
$\mathrm{SU}(N)_{\mathrm{f}}$:	unitary group describing N flavours
$\mathrm{SU}(3)_{\mathrm{c}}$:	colour group of quarks and gluons
$\Theta = \boldsymbol{PCT}$:	combined operation of space reflection, charge conjugation and time reversal
θ_{W}:	Weinberg, or weak interaction, angle. Note that only $\sin^2\theta_{\mathrm{w}}$ is physical
T:	time-reversal operation
T:	scattering matrix which enters cross-section formulae.
$T_k, k = 1, \ldots N$:	abstract generators of Lie group G, with N the dimension of the Lie algebra *Lie*(G)
$U(g)$:	unitary representation of group element $g \in G$ in a given representation space
$U(T_k)$:	representation of the generator T_k in a given representation space
V_{CKM}:	mixing matrix of *down*-type quarks
$\upsilon_\alpha(x)$:	weak vector current