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Option Theory with Stochastic Analysis

An Introduction to
Mathematical Finance

Springer

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To my wife Jūratė

Preface

Since 1972 and the appearance of the famous Black & Scholes option pricing formula, derivatives have become an integrated part of everyday life in the financial industry. Options and derivatives are tools to control risk exposure, and used in the strategies of investors speculating in markets like fixed-income, stocks, currencies, commodities and energy.

A combination of mathematical and economical reasoning is used to find the price of a derivatives contract. This book gives an introduction to the theory of *mathematical finance*, which is the modern approach to analyse options and derivatives. Roughly speaking, we can divide mathematical finance into three main directions. In *stochastic finance* the purpose is to use economic theory with stochastic analysis to derive fair prices for options and derivatives. The results are based on stochastic modelling of financial assets, which is the field of *empirical finance*. Numerical approaches for finding prices of options are studied in *computational finance*. All three directions are presented in this book. Algorithms and code for Visual Basic functions are included in the numerical chapter to inspire the reader to test out the theory in practice.

The objective of the book is *not* to give a complete account of option theory, but rather relax the mathematical rigour to focus on the ideas and techniques. Instead of going deep into stochastic analysis, we present the intuition behind basic concepts like the Itô formula and stochastic integration, enabling the reader to use these in the context of option theory. To comprehend the theory, a background in mathematics and statistics at bachelor level (that means, calculus, linear algebra and probability theory) is recommended.

This book is a revision of the Norwegian edition which appeared in 2001. It is used in a course for students at the University of Oslo preparing for a master in finance and insurance mathematics. The manuscript for the Norwegian edition grew from lecture notes prepared for an introductory course in modern finance for the industry.

Several people have contributed in the writing of this book. I am grateful to Jūratė Šaltytė-Benth for carefully reading through the manuscript and significantly improving the presentation, and Neil Shephard for providing me with `Ox` software to do statistical analysis of financial time series. Furthermore, the advice given and corrections made by Jeffrey Boys, Daniela Brandt,

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Oslo, August 2003 *Fred Espen Benth*

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