NONLINEAR PHYSICAL SCIENCE

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Nonlinear Physical Science focuses on recent advances of fundamental theories and principles, analytical and symbolic approaches, as well as computational techniques in nonlinear physical science and nonlinear mathematics with engineering applications.

Topics of interest in *Nonlinear Physical Science* include but are not limited to:

- New findings and discoveries in nonlinear physics and mathematics
- Nonlinearity, complexity and mathematical structures in nonlinear physics
- Nonlinear phenomena and observations in nature and engineering
- Computational methods and theories in complex systems
- Lie group analysis, new theories and principles in mathematical modeling
- Stability, bifurcation, chaos and fractals in physical science and engineering
- Nonlinear chemical and biological physics
- Discontinuity, synchronization and natural complexity in the physical sciences

Michal Fečkan

Bifurcation and Chaos in Discontinuous and Continuous Systems

With 30 figures

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Author

Michal Fečkan
Department of Mathematical Analysis and Numerical Mathematics
Faculty of Mathematics, Physics and Informatics, Comenius University
Mlynská dolina, 842 48 Bratislava, Slovakia
E-mail: Michal.Feckan@fmph.uniba.sk

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To my beloved family

Preface

This book is devoted to the comprehensive bifurcation theory of chaos in nonlinear dynamical systems with applications to mechanics and vibrations. Precise and complete proofs of derived mathematical results are presented with many stimulating and illustrative examples. I study bifurcations of chaotic solutions for perturbed problems from either homoclinic or heteroclinic orbits of unperturbed ones. This method is also known as the Melnikov-type approach. Certainly there are many interesting books in this direction, but all results of this book have not yet been published in any book, since I have collected some results of mine together with my coauthors appeared only in articles and manuscripts. So I hope that this book is a useful contribution to a rapidly developing theory of chaos and it is a good continuation of my recently published book in Springer with similar topics.

The book is intended to be used by scientists interested in the theory of chaos and its applications, like mathematicians, physicists, or engineers. It can also serve as a textbook for a class of nonlinear oscillations and dynamical systems.

Here is a brief outline of each chapter.

Chapter 1 is an introduction to the topic of the book by presenting two well-known chaotic models: damped and driven Duffing and pendulum equations.

To make this book as self-contained as possible, some basic preliminary results are included in Chapter 2.

Chapter 3 studies chaotic bifurcations of discrete dynamical systems including: nonautonomous difference equations; diffeomorphisms; perturbed singular and singularly perturbed impulsive ordinary differential equations (ODEs); and inflated dynamical systems arising in computer assisted proofs and in other numerical methods in dynamical systems, so an extension of Smale horseshoe to inflated dynamical systems is presented.

Chapter 4 deals with proving chaos for parameterized ODEs in arbitrary dimensions. It is shown that if the Melnikov function is identically zero the second order Melnikov function must be derived. I consider a broad variety of ODEs: coupled nonresonant ODEs, resonant systems of ODEs investigated with the help of averaging theory; singularly perturbed ODEs; and inflated ODEs. I also show that the structure of chaotic parameters is related to the Morin singularity of smooth mappings. I end this chapter with infinite dimensional ODEs on lattices by considering a model of two one-dimensional interacting sublattices of harmonically coupled protons and heavy ions.

Chapter 5 shows chaotic vibrations of partial differential equations (PDEs): slowly periodically perturbed and weakly nonlinear beams on elastic bearings; periodically forced and nonresonant buckled elastic beams; and periodically forced compressed beams at resonance.

Chapter 6 is devoted to the study of chaotic oscillations of discontinuous (nonsmooth) differential equations (DDEs). First I consider the case when the homoclinic orbit of the unperturbed DDE transversally crosses discontinuity surfaces. Then I study a chaos for time-perturbed DDEs. I apply our general results to quasiperiodic piecewise linear systems in $\mathbb{R}^3$, and to piecewise smooth forced planar DDEs. Then I extend those result to sliding homoclinic bifurcations, when a part of the homoclinic orbit of the unperturbed DDE lies on a discontinuity surface. A rigorous proof of the existence of chaos for stick-slip systems is presented. I utilize general theoretical results to planar and 3-dimensional sliding homoclinic cases.

In Chapter 7, first I investigate the Melnikov function in general by computing its Fourier coefficients. These computations allow me to find examples when the Melnikov function is ether identically zero or not. I also derive the second order Melnikov function when the (first order) Melnikov function is identically zero. For construction of concrete examples, I solve an inverse problem when the homoclinic orbit is given and a second order ODE is found so that it possesses that homoclinic orbit. The second part of this chapter is devoted to showing chaos near transversal heteroclinic orbits. The third part deals with the blue sky catastrophe for periodic orbits.

In all chapters, derived bifurcation conditions for the existence of chaos are expressed as simple zeroes of corresponding Melnikov functions. Functional analytic approaches are used which are roughly based on a concept of exponential dichotomy together with Lyapunov-Schmidt method. Numerical computations described by figures are given with the help of a computational software program *Mathematica*.

The author is indebted to the coauthors for some results mentioned in this book: Jan Awrejcewicz, Flaviano Battelli, Giovanni Colombo, Matteo Franca, Barnabás M. Garay, Joseph Gruendler, Paweł Olejnik, Weiyao Zeng. Partial support of Grants VEGA-SAV 2/0124/10, VEGA-MS 1/0098/08, an award from Literárny fond and by the Slovak Research and Development Agency under the contract No. APVV-0414-07 are also appreciated.

Michal Fečkan
Bratislava, Slovakia
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