

NONLINEAR PHYSICAL SCIENCE

NONLINEAR PHYSICAL SCIENCE

Nonlinear Physical Science focuses on recent advances of fundamental theories and principles, analytical and symbolic approaches, as well as computational techniques in nonlinear physical science and nonlinear mathematics with engineering applications.

Topics of interest in *Nonlinear Physical Science* include but are not limited to:

- New findings and discoveries in nonlinear physics and mathematics
- Nonlinearity, complexity and mathematical structures in nonlinear physics
- Nonlinear phenomena and observations in nature and engineering
- Computational methods and theories in complex systems
- Lie group analysis, new theories and principles in mathematical modeling
- Stability, bifurcation, chaos and fractals in physical science and engineering
- Nonlinear chemical and biological physics
- Discontinuity, synchronization and natural complexity in the physical sciences

SERIES EDITORS

Albert C.J. Luo

Department of Mechanical and Industrial
Engineering
Southern Illinois University Edwardsville
Edwardsville, IL 62026-1805, USA
Email: aluo@siue.edu

Nail H. Ibragimov

Department of Mathematics and Science
Blekinge Institute of Technology
S-371 79 Karlskrona, Sweden
Email: nib@bth.se

INTERNATIONAL ADVISORY BOARD

Ping Ao, University of Washington, USA; Email: aoping@u.washington.edu

Jan Awrejcewicz, The Technical University of Lodz, Poland; Email: awrejcew@p.lodz.pl

Eugene Benilov, University of Limerick, Ireland; Email: Eugene.Benilov@ul.ie

Eshel Ben-Jacob, Tel Aviv University, Israel; Email: eshel@tamar.tau.ac.il

Maurice Courbage, Université Paris 7, France; Email: maurice.courbage@univ-paris-diderot.fr

Marian Gidea, Northeastern Illinois University, USA; Email: mgidea@neiu.edu

James A. Glazier, Indiana University, USA; Email: glazier@indiana.edu

Shijun Liao, Shanghai Jiaotong University, China; Email: sjliao@sjtu.edu.cn

Jose Antonio Tenreiro Machado, ISEP-Institute of Engineering of Porto, Portugal; Email: jtm@dee.isep.ipp.pt

Nikolai A. Magnitskii, Russian Academy of Sciences, Russia; Email: nmag@isa.ru

Josep J. Masdemont, Universitat Politècnica de Catalunya (UPC), Spain; Email: josep@barquins.upc.edu

Dmitry E. Pelinovsky, McMaster University, Canada; Email: dmpeli@math.mcmaster.ca

Sergey Prants, V.I.I'ichev Pacific Oceanological Institute of the Russian Academy of Sciences, Russia;
Email: prants@poi.dvo.ru

Victor I. Shrira, Keele University, UK; Email: v.i.shrira@keele.ac.uk

Jian Qiao Sun, University of California, USA; Email: jqsun@ucmerced.edu

Abdul-Majid Wazwaz, Saint Xavier University, USA; Email: wazwaz@sxu.edu

Pei Yu, The University of Western Ontario, Canada; Email: pyu@uwo.ca

Michal Fečkan

Bifurcation and Chaos in Discontinuous and Continuous Systems

With 30 figures



Author

Michal Fečkan

Department of Mathematical Analysis and Numerical Mathematics

Faculty of Mathematics, Physics and Informatics, Comenius University

Mlynská dolina, 842 48 Bratislava, Slovakia

E-mail: Michal.Feckan@fmph.uniba.sk

ISSN 1867-8440

e-ISSN 1867-8459

Nonlinear Physical Science

ISBN 978-7-04-031533-2

Higher Education Press, Beijing

ISBN 978-3-642-18268-6

ISBN 978-3-642-18269-3 (eBook)

Springer Heidelberg Dordrecht London New York

Library of Congress Control Number: 2011920974

© Higher Education Press, Beijing and Springer-Verlag Berlin Heidelberg 2011

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable to prosecution under the German Copyright Law.

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

To my beloved family

Preface

This book is devoted to the comprehensive bifurcation theory of chaos in nonlinear dynamical systems with applications to mechanics and vibrations. Precise and complete proofs of derived mathematical results are presented with many stimulating and illustrative examples. I study bifurcations of chaotic solutions for perturbed problems from either homoclinic or heteroclinic orbits of unperturbed ones. This method is also known as the Melnikov-type approach. Certainly there are many interesting books in this direction, but all results of this book have not yet been published in any book, since I have collected some results of mine together with my coauthors appeared only in articles and manuscripts. So I hope that this book is a useful contribution to a rapidly developing theory of chaos and it is a good continuation of my recently published book in Springer with similar topics.

The book is intended to be used by scientists interested in the theory of chaos and its applications, like mathematicians, physicists, or engineers. It can also serve as a textbook for a class of nonlinear oscillations and dynamical systems.

Here is a brief outline of each chapter.

Chapter 1 is an introduction to the topic of the book by presenting two well-known chaotic models: damped and driven Duffing and pendulum equations.

To make this book as self-contained as possible, some basic preliminary results are included in Chapter 2.

Chapter 3 studies chaotic bifurcations of discrete dynamical systems including: nonautonomous difference equations; diffeomorphisms; perturbed singular and singularly perturbed impulsive ordinary differential equations (ODEs); and inflated dynamical systems arising in computer assisted proofs and in other numerical methods in dynamical systems, so an extension of Smale horseshoe to inflated dynamical systems is presented.

Chapter 4 deals with proving chaos for parameterized ODEs in arbitrary dimensions. It is shown that if the Melnikov function is identically zero the second order Melnikov function must be derived. I consider a broad variety of ODEs: coupled nonresonant ODEs, resonant systems of ODEs investigated with the help of averaging theory; singularly perturbed ODEs; and inflated ODEs. I also show that the structure of chaotic parameters is related to the Morin singularity of smooth map-

pings. I end this chapter with infinite dimensional ODEs on lattices by considering a model of two one-dimensional interacting sublattices of harmonically coupled protons and heavy ions.

Chapter 5 shows chaotic vibrations of partial differential equations (PDEs): slowly periodically perturbed and weakly nonlinear beams on elastic bearings; periodically forced and nonresonant buckled elastic beams; and periodically forced compressed beams at resonance.

Chapter 6 is devoted to the study of chaotic oscillations of discontinuous (non-smooth) differential equations (DDEs). First I consider the case when the homoclinic orbit of the unperturbed DDE transversally crosses discontinuity surfaces. Then I study a chaos for time-perturbed DDEs. I apply our general results to quasiperiodic piecewise linear systems in \mathbb{R}^3 , and to piecewise smooth forced planar DDEs. Then I extend those result to sliding homoclinic bifurcations, when a part of the homoclinic orbit of the unperturbed DDE lies on a discontinuity surface. A rigorous proof of the existence of chaos for stick-slip systems is presented. I utilize general theoretical results to planar and 3-dimensional sliding homoclinic cases.

In Chapter 7, first I investigate the Melnikov function in general by computing its Fourier coefficients. These computations allow me to find examples when the Melnikov function is either identically zero or not. I also derive the second order Melnikov function when the (first order) Melnikov function is identically zero. For construction of concrete examples, I solve an inverse problem when the homoclinic orbit is given and a second order ODE is found so that it possesses that homoclinic orbit. The second part of this chapter is devoted to showing chaos near transversal heteroclinic orbits. The third part deals with the blue sky catastrophe for periodic orbits.

In all chapters, derived bifurcation conditions for the existence of chaos are expressed as simple zeroes of corresponding Melnikov functions. Functional analytic approaches are used which are roughly based on a concept of exponential dichotomy together with Lyapunov-Schmidt method. Numerical computations described by figures are given with the help of a computational software program *Mathematica*.

The author is indebted to the coauthors for some results mentioned in this book: Jan Awrejcewicz, Flaviano Battelli, Giovanni Colombo, Matteo Franca, Barnabás M. Garay, Joseph Gruendler, Paweł Olejnik, Weiyao Zeng. Partial support of Grants VEGA-SAV 2/0124/10, VEGA-MS 1/0098/08, an award from Literárny fond and by the Slovak Research and Development Agency under the contract No. APVV-0414-07 are also appreciated.

Michal Fečkan
Bratislava, Slovakia
June 2010

Contents

1	Introduction	1
	References	6
2	Preliminary Results	9
2.1	Linear Functional Analysis	9
2.2	Nonlinear Functional Analysis	11
2.2.1	Banach Fixed Point Theorem	11
2.2.2	Implicit Function Theorem	11
2.2.3	Lyapunov-Schmidt Method	12
2.2.4	Brouwer Degree	13
2.2.5	Local Invertibility	13
2.2.6	Global Invertibility	14
2.3	Multivalued Mappings	14
2.4	Differential Topology	15
2.4.1	Differentiable Manifolds	15
2.4.2	Vector Bundles	16
2.4.3	Tubular Neighbourhoods	16
2.5	Dynamical Systems	17
2.5.1	Homogenous Linear Equations	17
2.5.2	Chaos in Diffeomorphisms	18
2.5.3	Periodic ODEs	19
2.5.4	Vector Fields	20
2.5.5	Global Center Manifolds	22
2.5.6	Two-Dimensional Flows	22
2.5.7	Averaging Method	23
2.5.8	Carathéodory Type ODEs	24
2.6	Singularities of Smooth Maps	24
2.6.1	Jet Bundles	24
2.6.2	Whitney C^∞ Topology	25
2.6.3	Transversality	25
2.6.4	Malgrange Preparation Theorem	26

2.6.5	Complex Analysis	26
References	28
3	Chaos in Discrete Dynamical Systems	29
3.1	Transversal Bounded Solutions	29
3.1.1	Difference Equations	29
3.1.2	Variational Equation	30
3.1.3	Perturbation Theory	35
3.1.4	Bifurcation from a Manifold of Homoclinic Solutions.....	38
3.1.5	Applications to Impulsive Differential Equations	40
3.2	Transversal Homoclinic Orbits	44
3.2.1	Higher Dimensional Difference Equations	44
3.2.2	Bifurcation Result	45
3.2.3	Applications to McMillan Type Mappings	51
3.2.4	Planar Integrable Maps with Separatrices	54
3.3	Singular Impulsive ODEs	55
3.3.1	Singular ODEs with Impulses	55
3.3.2	Linear Singular ODEs with Impulses	56
3.3.3	Derivation of the Melnikov Function	64
3.3.4	Examples of Singular Impulsive ODEs	68
3.4	Singularly Perturbed Impulsive ODEs	70
3.4.1	Singularly Perturbed ODEs with Impulses	70
3.4.2	Melnikov Function	71
3.4.3	Second Order Singularly Perturbed ODEs with Impulses ...	72
3.5	Inflated Deterministic Chaos	73
3.5.1	Inflated Dynamical Systems	73
3.5.2	Inflated Chaos	74
References	83
4	Chaos in Ordinary Differential Equations	87
4.1	Higher Dimensional ODEs	87
4.1.1	Parameterized Higher Dimensional ODEs	87
4.1.2	Variational Equations	88
4.1.3	Melnikov Mappings	90
4.1.4	The Second Order Melnikov Function	93
4.1.5	Application to Periodically Perturbed ODEs	95
4.2	ODEs with Nonresonant Center Manifolds	97
4.2.1	Parameterized Coupled Oscillators	97
4.2.2	Chaotic Dynamics on the Hyperbolic Subspace	98
4.2.3	Chaos in the Full Equation	100
4.2.4	Applications to Nonlinear ODEs	105
4.3	ODEs with Resonant Center Manifolds	108
4.3.1	ODEs with Saddle-Center Parts	108
4.3.2	Example of Coupled Oscillators at Resonance	109
4.3.3	General Equations	121

- 4.3.4 Averaging Method 127
- 4.4 Singularly Perturbed and Forced ODEs 131
 - 4.4.1 Forced Singular ODEs 131
 - 4.4.2 Center Manifold Reduction 132
 - 4.4.3 ODEs with Normal and Slow Variables 135
 - 4.4.4 Homoclinic Hopf Bifurcation 135
- 4.5 Bifurcation from Degenerate Homoclinics 136
 - 4.5.1 Periodically Forced ODEs with Degenerate Homoclinics . . . 136
 - 4.5.2 Bifurcation Equation 137
 - 4.5.3 Bifurcation for 2-Parametric Systems 138
 - 4.5.4 Bifurcation for 4-Parametric Systems 144
 - 4.5.5 Autonomous Perturbations 147
- 4.6 Inflated ODEs 150
 - 4.6.1 Inflated Carathéodory Type ODEs 150
 - 4.6.2 Inflated Periodic ODEs 151
 - 4.6.3 Inflated Autonomous ODEs 154
- 4.7 Nonlinear Diatomic Lattices 156
 - 4.7.1 Forced and Coupled Nonlinear Lattices 156
 - 4.7.2 Spatially Localized Chaos 157
- References 163

- 5 Chaos in Partial Differential Equations 167**
 - 5.1 Beams on Elastic Bearings 167
 - 5.1.1 Weakly Nonlinear Beam Equation 167
 - 5.1.2 Setting of the Problem 168
 - 5.1.3 Preliminary Results 171
 - 5.1.4 Chaotic Solutions 191
 - 5.1.5 Useful Numerical Estimates 215
 - 5.1.6 Lipschitz Continuity 217
 - 5.2 Infinite Dimensional Non-Resonant Systems 220
 - 5.2.1 Buckled Elastic Beam 220
 - 5.2.2 Abstract Problem 224
 - 5.2.3 Chaos on the Hyperbolic Subspace 224
 - 5.2.4 Chaos in the Full Equation 226
 - 5.2.5 Applications to Vibrating Elastic Beams 227
 - 5.2.6 Planer Motion with One Buckled Mode 227
 - 5.2.7 Nonplaner Symmetric Beams 230
 - 5.2.8 Nonplaner Nonsymmetric Beams 235
 - 5.2.9 Multiple Buckled Modes 238
 - 5.3 Periodically Forced Compressed Beam 242
 - 5.3.1 Resonant Compressed Equation 242
 - 5.3.2 Formulation of Weak Solutions 242
 - 5.3.3 Chaotic Solutions 243
- References 247

6	Chaos in Discontinuous Differential Equations	249
6.1	Transversal Homoclinic Bifurcation	249
6.1.1	Discontinuous Differential Equations	249
6.1.2	Setting of the Problem	250
6.1.3	Geometric Interpretation of Nondegeneracy Condition	255
6.1.4	Orbits Close to the Lower Homoclinic Branches	257
6.1.5	Orbits Close to the Upper Homoclinic Branch	263
6.1.6	Bifurcation Equation	265
6.1.7	Chaotic Behaviour	287
6.1.8	Almost and Quasiperiodic Cases	293
6.1.9	Periodic Case	294
6.1.10	Piecewise Smooth Planar Systems	295
6.1.11	3D Quasiperiodic Piecewise Linear Systems	299
6.1.12	Multiple Transversal Crossings	310
6.2	Sliding Homoclinic Bifurcation	312
6.2.1	Higher Dimensional Sliding Homoclinics	312
6.2.2	Planar Sliding Homoclinics	319
6.2.3	Three-Dimensional Sliding Homoclinics	321
6.3	Outlook	332
	References	332
7	Concluding Related Topics	335
7.1	Notes on Melnikov Function	335
7.1.1	Role of Melnikov Function	335
7.1.2	Melnikov Function and Calculus of Residues	336
7.1.3	Second Order ODEs	340
7.1.4	Applications and Examples	347
7.2	Transverse Heteroclinic Cycles	361
7.3	Blue Sky Catastrophes	369
7.3.1	Symmetric Systems with First Integrals	370
7.3.2	D'Alembert and Penalized Equations	371
	References	373
	Index	375