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Early Algebraization

A Global Dialogue from Multiple Perspectives

Springer

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Foreword

Early Algebraization: A Global Dialogue from Multiple Perspectives is the second monograph in the Advances in Mathematics Education (AiME) series launched by Springer in 2009. The book follows in the tradition of *Theories of Mathematics Education* (Sriraman and English, monograph 1), stemming from a previous ZDM issues on early algebraic thinking (vol. 37, no. 1, 2005 and vol. 40, no. 1, 2008). That is, although it uses the previous issues as a basis for the current monograph, the monograph itself goes beyond simply revisiting the past. It conveys the present state of the art on existing research on early algebraization since 2005. The eight previous articles (five from vol. 37 and 3 from vol. 40) have been reworked and updated in addition to 18 new chapters from researchers involved in early algebraization research projects in different parts of the world, which include 4 commentaries on the scope of the research.

The book editors Jinfa Cai and Eric Knuth have compiled the book in three substantial parts between the bookends of a general introduction and an overall commentary addressing perspectives for research and teaching in this domain of inquiry. These three parts of the book examine curricular, cognitive and instructional components of early algebraization. Unlike the ZDM issue which was predominantly articles from researchers based in North America, this book contains ongoing research from different parts of the world, and initiates a global conversation on where the community stands in its research findings.

AiME is distinct from other mathematics education series because it attempts to draw the reader into a conversation, and be dialogic in its presentation. This is the purpose of soliciting commentaries from those that are able to synthesize ideas, expose them in a larger light of what is known, and directions in which they can be further pushed. This book continues in this tradition and attempts to draw us into the issues of understanding, implementing and assessing early algebraization in projects and curricula in different parts of the world. We hope this monograph is of value to the research community of mathematics educators interested in the role and significance of early algebraic thinking within the current research architecture. We appreciate the efforts of the authors that are a part of this book and thank the book editors (Cai & Knuth) for this superb book.

Gabriele Kaiser
Bharath Sriraman

Introduction

A Global Dialogue About Early Algebraization from Multiple Perspectives

Kilpatrick and Izsák (2008) quoted an anonymous editorial writer to start their chapter in the National Council of Teachers of Mathematics' 70th Yearbook: "If there is a heaven for school subjects, algebra will never go there. It is the one subject in the curriculum that has kept children from finishing high school, from developing their special interests and from enjoying much of their home study work. It has caused more family rows, more tears, more heartaches, and more sleepless nights than any other school subject." (p. 3) Even though there has been a dramatic change for the world 70 years ago when the editorial was written to nowadays, the status of algebra as a school subject has not changed much—algebra is important but many students experience difficulties (Kieran 2007; Loveless 2008; National Mathematics Advisory Panel [NMAP] 2008). In fact, algebra has been characterized as the most important "gatekeeper" in school mathematics.

Given its gatekeeper role as well as growing concern about students' inadequate understandings and preparation in algebra, algebra curricula and instruction have become focal points for policy makers and mathematics education researchers around the world (e.g., Bednarz et al. 1996; Lacampagne et al. 1995; RAND Mathematics Study Panel 2003; Stacey et al. 2004). An important emphasis, common around the globe, is the development of students' algebraic thinking in earlier grades. The development of students' algebraic thinking in earlier grades is not a new idea; in China and Russia, for example, algebraic concepts were introduced to elementary school students in the 50s and 60s. In other countries (e.g., Europe, North America), the discussion of integrating algebraic ideas into mathematics curricula in the earlier grades started in the 70s. In the past decade, however, there has been an increased emphasis on and wider acceptance for developing students' algebraic ideas and thinking in earlier grades, reflected in a number of influential policy documents. For example, in the United States, the NCTM proposed algebra as a content strand for all grade levels (NCTM 2000). In fact, it is widely accepted that to achieve the goal of "algebra for all", students in elementary and middle school must have experiences that better prepare them for more formal study of algebra in the later grades. Yet, only recently have researchers started to explore issues related to early algebraization.

Although a chapter on research on school algebra appeared in the *Handbook of the Research on Mathematics Teaching and Learning* (Grouws 1992), its focus was primarily on algebra at the secondary school level. In the *Second Handbook of the Research on Mathematics Teaching and Learning* (Lester 2007), there is again a chapter on algebra at the secondary school level, however, the volume now also includes a chapter on early algebra learning. In fact, this is the only chapter with such a focus in mathematics education research handbooks published in the past two decades. A similar trend can also be seen in publications directed toward teachers: In 1993, NCTM published a volume focused on research ideas for the elementary school classroom that did not include any chapters focused on early algebra learning; in contrast, NCTM recently published a similar volume (Lambdin and Lester 2010) that does include a chapter on early algebra learning. On one hand, such changes suggest that the field has known enough about early algebraization to synthesize research findings in the area. On the other hand, as Carraher and Schliemann (2007) recently pointed out: "Although there is some agreement that algebra has a place in the elementary school curriculum, the research basis needed for integrating algebra into the early mathematics curriculum is still emerging, little known, and far from consolidated." (p. 671) In fact, curriculum developers, educational researchers, teachers, and policy makers are just beginning to think about and explore the kinds of mathematical experiences and knowledge students in early grades need to be successfully prepared for the formal study of algebra in the later grades. This monograph is part of such an effort.

Early Algebraization

Traditionally, most school mathematics curricula separate the study of arithmetic and algebra—arithmetic being the primary focus of elementary school mathematics and algebra the primary focus of middle and high school mathematics. There is a growing consensus, however, that this separation makes it more difficult for students to learn algebra in the later grades (Kieran 2007). Moreover, based on recent research on learning, there are many obvious and widely accepted reasons for developing algebraic ideas in the earlier grades (Cai and Knuth 2005). The field has gradually reached consensus that students can learn and should be exposed to algebraic ideas as they develop the computational proficiency emphasized in arithmetic. In addition, it is agreed that the means for developing algebraic ideas in earlier grades is not to simply push the traditional secondary school algebra curriculum down into the elementary school mathematics curriculum. Rather, developing algebraic ideas in the earlier grades requires fundamentally reforming how arithmetic should be viewed and taught as well as a better understanding of the various factors that make the transition from arithmetic to algebra difficult for students.

The transition from arithmetic to algebra is difficult for many students, even for those students who are quite proficient in arithmetic, as it often requires them to think in very different ways (Kieran 2007; Kilpatrick et al. 2001). Kieran, for example, suggested the following shifts from thinking arithmetically to thinking algebraically: (1) A focus on relations and not merely on the calculation of a numerical answer; (2) A focus on operations as well as their inverses, and on the related idea of doing/undoing; (3) A focus on both representing and solving a problem rather than on merely solving it; (4) A focus on both numbers and letters, rather than on numbers alone; and (5) A refocusing of the meaning of the equal sign from a signifier to calculate to a symbol that denotes an equivalence relationship between quantities. These five shifts certainly fall within the domain of arithmetic, yet, they also represent a movement toward developing ideas fundamental to the study of algebra. Thus, in this view, the boundary between arithmetic and algebra is not as distinct as often is believed to be the case.

What is algebraic thinking in earlier grades then? Algebraic thinking in earlier grades should go beyond mastery of arithmetic and computational fluency to attend to the deeper underlying structure of mathematics. The development of algebraic thinking in the earlier grades requires the development of particular ways of thinking, including analyzing relationships between quantities, noticing structure, studying change, generalizing, problem solving, modeling, justifying, proving, and predicting. That is, early algebra learning develops not only new tools to understand mathematical relationships, but also new habits of mind. In this volume, we focus on the development of algebraic ideas in both elementary and middle schools.

Multiple Perspectives

In this volume, the authors address the issues of early algebraization from curricular, cognitive, and instructional perspectives. The inclusion of middle grades is desirable because of the critical transition from elementary to the middle grades, particularly related to algebra learning. The inclusion of issues related to curriculum, cognition, and instruction is based on the consideration that they are the three most fundamental perspectives for mathematics education. Curricula have a significant influence on what students learn (NCTM 2000) and have been found to contribute to mathematical performance differences in cross-national studies (Schmidt et al. 1996). Accordingly, the examination of curricula from various nations can provide a broader point of view regarding curricular approaches to integrating algebraic ideas into earlier grades as well as providing insights regarding the development of students' algebraic thinking.

Although curricula can provide elementary and middle school students with opportunities to develop their algebraic thinking, teachers are arguably the most important influence on what students actually learn. Thus, the success of efforts to develop students' algebraic thinking rests largely with the ability of teachers to foster such thinking.

The design of curricula and professional development programs as well as the enactment of instructional practices intended to support the development of students' algebraic thinking are all dependent, to a great extent, on what we know about students' algebraic thinking and its development. Thus it is critical to examine issues related to students' cognition in algebra learning.

As we look across this set of articles in this volume, with their variety of foci and perspectives, two cross-cutting themes surfaced. First is the importance of better integrating into current school mathematics practices opportunities for students to develop their algebraic thinking. These opportunities include both the design of curricula, at the elementary school level in particular, that pays explicit attention to making connections between arithmetic and algebra, and the recognition of opportunities to strengthen these connections as students progress through middle school. The second theme to emerge is the importance of supporting teachers' efforts to implement practices that foster the development of students' algebraic thinking. If future generations of students are to become better prepared for more formal study of algebra in the later grades, then likewise teachers must also be better prepared. The articles in this volume provide guidance and suggestions for continued work in the area of early algebra research regarding teachers' instructional practices and professional development.

One of the important features of this volume is its international in nature, which promotes a global dialogue on the topic. Research is presented from many parts of the world, including Australia, Canada, China, France, India, Italy, Japan, New Zealand, Russia, Singapore, South Korea, the United Kingdom, and the United States of America. Such a global dialogue will help us address issues related to early algebra learning and, ultimately, better prepare greater numbers of students for success in algebra.

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References

Bednarz, N., Kieran, C., & Lee, L. (Eds.) (1996). *Approaches to Algebra: Perspectives for Research and Teaching*. Dordrecht: Kluwer Academic Publishers.

Cai, J., & Knuth, E. (Eds.) (2005). Developing algebraic thinking: Multiple perspectives. *Zentralblatt fuer Didaktik der Mathematik (International Journal on Mathematics Education)*, *37*(1). Special Issue.

Carraher, D. W., & Schliemann, A. D. (2007). Early algebra and algebraic reasoning. In F. K. Lester Jr. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics* (pp. 669–706). Charlotte, NC: Information Age Publishing; Reston, VA: National Council of Teachers of Mathematics.

Chick, E., Stacey, K., Vincent, J., & Vincent, J. (Eds.) (2001). *The Future of the Teaching and Learning of Algebra: Proceedings of 12th ICMI Study*. University Melbourne, Australia.

Grouws, D. A. (Ed.) (1992). *Handbook of Research on Mathematics Teaching and Learning*. Reston, VA: National Council of Teachers of Mathematics.

Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels. In F.K. Lester Jr. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning*. Charlotte, NC: New Age Publishing; Reston, VA: National Council of Teachers of Mathematics.

Kilpatrick, J., & Izsák, A. (2008). A history of algebra in the school curriculum. In C. E. Greenes & R. Rubenstein (Eds.), *Algebra and Algebraic Thinking in School Mathematics. 70th Yearbook of the National Council of Teachers of Mathematics* (pp. 3–18). Reston, VA: National Council of Teachers of Mathematics.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.) (2001). *Adding it up*. Washington, DC: National Academy Press.

Lacampagne, C., Blair, W., & Kaput, J. (1995). *The Algebra Initiative Colloquium*. Washington, DC: U.S. Department of Education & Office of Educational Research and Improvement.

Lambdin, D. V., & Lester, F. K. Jr. (Eds.) (2010). *Teaching and Learning Mathematics: Translating Research for Elementary School Teachers*. Reston, VA: National Council of Teachers of Mathematics.

Lester, F. K. Jr. (Ed.) (2007). *Second Handbook of Research on Mathematics Teaching and Learning*. Charlotte, NC: Information Age Publishing; Reston, VA: National Council of Teachers of Mathematics.

Loveless, T. (2008). *The misplaced math student: lost in eight-grade algebra*. The Brown Center Report on American Education.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

National Mathematics Advisory Panel (2008). *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

RAND Mathematics Study Panel (2003). *Mathematical proficiency for all students: Towards a strategic development program in mathematics education*. Santa Monica, CA: RAND Corporation MR-1643-OERI.

Schmidt, W. H., McKnight, C. C., & Raizen, S. A. (1996). *A Splintered Vision: An Investigation of U.S. Science and Mathematics Education*. Boston: Kluwer.

Stacey, K., Chick, H., & Kendal, M. (Eds.) (2004). *The Future of the Teaching and Learning of Algebra: The 12th ICMI Study*. Kluwer: Dordrecht.

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