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Paul Malliavin

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Preface

An appropriate coverage of the subjects contained in the five parts of this book would need several monographs. We hope that the global treatment presented here may emphasize some of their deep interactions. As far as possible we present self-contained proofs; we have also tried to produce a book that could be used in a graduate course.

Our thread of Ariadne is the introduction into stochastic analysis of the methodology used in classical analysis and differential geometry. Our geometric point of view has obliged us to pay great attention to the foundations. On the other hand our notation, which follows the usual conventions, will allow an experienced worker to look directly at any section of this book, without spending time on the foundational sections.

Each part is constructed according to the following format: a short introduction, a detailed table of contents at the beginning of each chapter of that part and a short note on the literature at the end of each part.

The style of writing oscillates from one part to the next between that of a rather technical monograph in Part II to a broader survey style in Parts IV and V.

I owe a great debt to K. Itô, J.R. Norris, D.W. Stroock for their careful reading of the first draft and for their far-reaching suggestions.

Paris, January 1997 *Paul Malliavin*

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