Part II
Microscopic Description of Emission Processes

The most general framework to describe emission processes is, of course, the time dependent approach. In this part of the book, devoted to microscopic approaches, we will derive the general expression of the decay width, known as the Fermi golden rule, by using time-dependent Schrödinger equation. Then, we will show that this relation can be recovered within various approaches like reaction theory or R-matrix approach. The R-matrix theory [1, 2] makes a step forward with respect to the Gamow theory [3], by expressing the decay width as a product between the particle preformation probability and the penetration through the barrier [4–9]. This relation is similar to its phenomenological counterpart (2.87), but the role of the wave function at the matching point $f_c(R)/R$ is played by the preformation amplitude, already introduced by Eq. 2.143 and defined as the overlap between the initial wave function and the product of the daughter and α-particle wave functions. This approach takes into account the nuclear structure details, by expressing the cluster wave function in terms of two proton and two neutron orbitals in some mean field, interacting with each other via two-body residual forces [5, 8]. We will describe the general procedure to estimate the preformation amplitude within the so-called Multi-step Shell Model technique (MSM) [10, 11]. Due to the antisymmetrization effects between the α-particle and daughter wave functions, the interaction becomes non-local in the internal region [12, 13].

It was shown that the usual shell model space using $N = 6$–8 major shells underestimates the experimental decay width by several orders of magnitude [14, 15], due to the exponential decrease of bound single particle wave functions [16]. An answer to the problem would be the inclusion of the sp narrow resonances lying in continuum [17–19], i.e. Gamow states. In spite of the fact that the true asymptotic behaviour of the wave functions is achieved, the value of the half life is still not reproduced [20]. Only the background components in continuum can describe the right order of magnitude of experimental decay widths [21–25]. The inclusion of the background contribution becomes important because an important part of the α-clustering process proceeds through such states.

The problem of considering the continuum part of the spectrum in microscopic calculations is rather involved, but very important especially for drip line nuclei [26]. The idea to replace the integration over the real spectrum in continuum by sp Gamow resonances plus an integration along a contour in the complex plane including these resonances was considered by Berggren in Ref. [27]. The calculation is very much simplified if one considers that in some physical processes only the narrow resonances are relevant and the integration, giving the background, can be neglected [28, 29]. This was shown to be an adequate approach, for instance in giant resonances [30] and in the nucleon decay processes [31]. To estimate the decay width, the states in continuum can be taken into account effectively by including a cluster component [32], or by considering a sp basis with a larger harmonic oscillator (ho) parameter for states in continuum [25].

The α-decaying state can be described as a sp resonance, namely by using the matching between logarithmic derivatives of the preformation amplitude and external Coulomb function. The derivative of the α-particle preformation factor, estimated within the shell model is almost constant along any neutron chain and therefore is not consistent with the decreasing behaviour of Q-values along such chains [33, 34]. We will show that the slope of the preformation amplitude can be corrected by changing the ho parameter of sp components. These components are related to an α-cluster term, not predicted by the standard shell model [32]. Indeed, the even–odd pair staggering of binding energies found along the α-lines with $N-Z=$ const., can be explained in terms of a "pairing" in the isospin space between proton and neutron pairs, considered as bosons [35, 36]. The generalization of this approach in terms many-body Greens's functions was performed in Ref. [37, 38]. This suggest that α-particles are already preformed at least in the low density region of the nuclear surface and they can explain the above inconsistency. We will discuss all these points within the so-called selfconsistent emission theory.

Finally we will describe the α-decay fine structure to vibrational states within the Quasiparticle Random Phase Approximation (QRPA) and the generalization of the preformation amplitude to heavy cluster decays. A brief description of fission-like theory within the Two Center Shell Model is also given.

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