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Julia Drechsel

# Cooperative Lot Sizing Games in Supply Chains

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# Preface

This book results from my work as a research assistant at the Universities of Freiberg and Duisburg-Essen. Starting this research was very challenging due to the highly mathematically coined literature regarding game theoretical topics. The literature shows that cooperative game theory is well studied from a theoretical point of view, but that fields of application are fairly limited up to now. Hence, the following research question arose: What can be done to make concepts from cooperative game theory more applicable to practical problems?

The field of supply chain management is predestined for many kinds of cooperation because companies will be more successful if they consider relationships to suppliers, customers, and/or competitors. Therefore, this work focuses on cooperations in supply networks (horizontal and vertical cooperation) concerning joint ordering and/or joint production.

Problems of ordering and production for single decision makers are well studied in operations research and are used as a basis to develop cooperative models that display situations of cooperative decision making. Being one of the most essential problems in cooperations, the allocation of joint profits or costs is the fundamental question in cooperative game theory and will be the basic question answered by this work as well.

In both places of my doctoral studies, Freiberg and Duisburg, I had valuable support without which this work would not have been possible. First of all, I thank Alf Kimms for providing me the chance to work on this topic, his guidance, challenging discussions, and the possibility to continue my work in Duisburg.

I express my gratitude to Sarah Bretschneider, Demet Çetiner, Michaela Graf, Klaus-Christian Maassen, and Michael Müller-Bungart for providing me such a great research and working atmosphere in Freiberg and Duisburg – all of you helped me through struggles with research, programming, or teaching, as well as sometimes non-work related things, which was always valuable and helpful and made these four years a wonderful time. Furthermore, I would like to thank Annette Hoffmann and Nadine Krumpholz for welcoming me in Duisburg, Stefanie Kockerols for providing me help with the literature research and Çağdaş Özgür for a constant LATEX support.

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Duisburg  
May 2010

*Julia Drechsel*

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