Power Systems

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Power System Modelling and Scripting

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To Yolanda and Alessandro

Τί δῆτα, ὦ ξένε, εἴδωλον ἄν φαῖμεν
εἴναι πλήν γε τὸ πρὸς τἀληχινόν
ἀφωμοιωμένον ἕτερον τοιοῦτον;

Plato, *Sophist*, 365-361 B.C.

2.1 We make ourselves pictures of facts.
2.12 The picture is a model of reality.
2.225 There is no picture which is a priori true.

Ludwig Wittgenstein, *Tractatus Logico-Philosophicus,* 1922 A.D.

Preface

History the Book

The first draft of these notes was born in the winter of 2002. At that time, I was a visiting scholar at the University of Waterloo. Originally, those notes were not intended as a book, but as a quick reference for not forgetting the models I was implementing for my research. After eight years, I am with Universidad de Castilla-La Mancha. During these years, the notes have been growing up little by little, ceaselessly. During the summer of 2009, I have reorganized the notes in the present book.

Justification of the Title

Power system modelling and scripting is a quite general and ambitious title. Of course, to embrace all existing aspects of power system modelling would lead to an encyclopedia. Thus, the book focuses on a subset of power system models based on the following assumptions: (i) devices are modelled as a set of nonlinear differential algebraic equations, (ii) all alternate-current devices are operating in three-phase balanced fundamental frequency, and (iii) the time frame of the dynamics of interest ranges from tenths to tens of seconds. These assumptions basically restrict the analysis to transient stability phenomena and generator controls. The modelling step is not self-sufficient. Mathematical models have to be translated into computer programming code in order to be analyzed, understood and "experienced". It is an object of the book to provide a general framework for a power system analysis software tool and hints for filling up this framework with versatile programming code.

Objectives of the Book

This book is for all students and researchers that are looking for a quick reference on power system models or need some guidelines for starting the challenging adventure of writing their own code. Thus, the objectives of this book are twofold.

The primary objective is to provide a selection of the most used device models ranging from static models for power flow, continuation power flow and optimal power flow analyses to as complete as possible dynamic electromechanical models for small-signal stability analysis and time domain simulations. This selection includes classical devices (e.g., synchronous machines) as well as non-conventional distributed energy resources (e.g., wind turbines), static voltage dependent loads as well as emerging energy storage devices. While describing each device, no matter if it is a well-known PV bus or a very specific pitch angle control for wind turbines, the focus is on the model hypotheses and on the implications of adopted simplifications.

The second objective is to provide a guide for organizing and translating mathematical models into computer programming code. The purpose is that the reader understands that there is always a gap between printed equations and software applications running on computers. Fortunately, this gap is not so huge and the book attempts to provide the methodological approach to fill it.

Choice of the Programming Language

When dealing with programming issues, one has to face and answer a tricky question: which is the most adequate computer language for tackling power system analysis? Then, after deciding on the language, one already knows that in a decade that language will be inevitably obsolete and a newer, easier, classier language will be available. To avoid a quick obsolescence, the goal of the book is not to provide code, but rather to teach how to design, organize and eventually write it. Programming issues will be always the same, at least as far as power systems will be the way they are. Thus, the adopted language is not so important.

At the end of a careful one-year-long study, I finally opted for the Python programming language. This language is well documented on the Internet, is elegant and neat, is fully based on classes and provides efficient libraries for solving linear algebra, handling sparse matrices and producing publication quality figures. Last but not least, the Python interpreter is free and open source. These characteristics do not guarantee that Python will last forever, but make it very appropriate for educational purposes.

Organization of the Book

The material included in this book is organized in a somewhat unorthodox way. Since the purpose is to concentrate on modelling, main power system analysis tools and basic programming concepts are introduced before describing the devices. The book is organized in five parts, as follows.

Part I contains introductory concepts. Chapter 1 provides the motivation of the book, some *philosophical* foundations of the art of modelling physical systems and defines the general mathematical model used for describing the behavior of power systems. Chapter 2 introduces the structure and the features of a software package for power system analysis while Chapter 3 discusses on the concept of *scripting* applied to power system analysis. The latter chapter also attempts to provide general guidelines for *thinking* power systems analysis in terms of computer programming. I hope that the results can be useful for Ph.D. students that, at the very end, will be the only readers of this book that have time to implement their own software applications.

Part II introduces basic tools for power system analysis. The viewpoint used for describing these tools is as general as possible. Chapter 4 describes the power flow analysis, Chapter 5 the continuation power flow, Chapter 6 the optimal power flow, Chapter 7 the small signal stability analysis, and Chapter 8 the time domain integration. Each topic is huge and, thus, only a very reduced selection of methods and algorithms is presented. The object is to provide a starting point for further investigations as well as a basement on top of which the following part dedicated to device modelling can be built.

Part III is the barycentric and most extended part of the book. It embraces the most important families of power system devices in an as systematic and exhaustive way as possible. Chapter 9 provides an introduction to the basic mathematical aspects of a generic electrical device. Following Chapters from 10 to 20 describe static power flow devices, transmission lines, static and regulating transformers, optimal power flow models, faults, protections, measurement devices, non-conforming static and dynamic loads, synchronous and induction machines, primary frequency and voltage regulators and power system stabilizers, dc devices, ac-dc devices, FACTS devices, and wind turbines and other distributed energy resources.

Part IV discusses spare topics that are relevant for power system analysis but are seldom included in power system books. Chapter 21 introduces the variegated world of data formats and discusses the challenges for creating a common model for exchanging power system data. Chapter 22 discusses the advantages of the Unix-style command line approach versus graphical user interfaces. Chapter 22 also describes plotting utilities aimed to power system visualization ranging from conventional plots to advanced 2D and 3D temperature maps. Chapter 23 describes some relevant educational aspects of free and open source power system software packages.

Finally Part V contains supporting material in form of appendices. Appendix A provides a minimal introduction to the Python non-standard scientific libraries used in the book. The aim of Appendix A is to make the book as self-contained as possible. Appendix B defines Python structures and classes that are used in the examples of the book. Appendix C discusses control diagrams and hard limit models. Finally, Appendix D provides the power system data used in the example of previous chapters whereas Appendix E describes the software requirements for working with the book as well as some useful links related to power system analysis.

Style of the Book

The style used in the book is somewhat unconventional with respect to traditional references about power system analysis. The will of merging together two worlds, namely power system modelling and computer programming for computational science, leads to the necessity of using a hybrid style that is unusual for both worlds. The major risk is perhaps to end up writing a software manual. To avoid that, I have tried to be as rigorous as possible and to make the examples based on computer code a supporting material rather than an essential part of the book, so that readers that despise computer code can skip it. I have also tried to apply the lesson of the Venikov's "Theory of Similarity and Simulation" [325]: whenever possible, I have included analogies and similarities taken from any mathematical and scientific field.

The material is organized in several parts, each part in several chapters and each chapter in several sections and subsections. This fragmentation can remind Seneca's style *arena sine calce* (i.e., sand without concrete) and is a kind of deformation due to the habit of object-oriented programming. However, this style is also dictated by the hope that in this way each topic can be easily found and fixed in mind.

For those interested in very technicalities, to write this book, I used LaTeX 3 with some useful packages such as PSfrag for the fine adjusting of figures and the IEEE style for formatting the bibliography data base. Python 2.6.2 was used as main environment while modules CVXOPT 1.1.2 and NumPy 1.3 were used for linear algebra, sparse matrix and eigenvalue analysis. Matplotlib 0.99 was used for generating simulation plots and Xfig 3.2.5 for drawing all other figures.

Acknowledgments

There is a beautiful Italian word that defines someone able to teach such that he changes someone else life and makes it irremediably better. This word is *maestro*. I have been lucky enough to have good ones: my grandfather Cesare, my father Guido and my mother Silvana, Profs. Bruno Delfino, Gio Battista Denegri and Marco Invernizzi from Università degli Studi di Genova, Prof. Claudio Cañizares from University of Waterloo and Prof. Antonio Conejo from Universidad de Castilla-La Mancha.

Concluding Remark

While completing this preface, I realize that much material has been left out of the book. However, I hope that what is included will be enough to transmit to the reader my passion for power system modelling and scripting. The book will accomplish its ultimate object if the next time the reader looks at some differential algebraic equations defining a power system device, he or she will be seized by a vague intellectual pleasure and a subtle ardent curiosity.

Waterloo, Genova, Ciudad Real 2002-2010

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Notation

To set up a complete list of symbols for a book of this kind is a complex task. The variety of models and physical quantities used in Part III leads to a huge list of symbols. On the other hand, Parts I and II present general mathematical concepts that require a careful notation consistency. Hence, two notation approaches are used throughout the book.

The first notation concerns common quantities and general mathematical functions and variables. This notation is common to the whole book and is followed as rigorously as possible.

The second notation concerns local parameters that are needed to describe each device model included in Part III. These parameters are defined the first time they appear or gathered in tables. The organization of the book in several chapters helps avoid confusion and allows keeping each device model or group of models well separated from the others.

General Notation Rules

General notation rules are as follows.

1. Scalar functions, variables and parameters expressed in pu are in lower case Latin fonts. For example: x, v, p.
2. Angles and angular speeds are given in lower case Greek fonts. For example: θ, δ, α.
3. Upper case Latin fonts indicate scalar variables and parameters expressed in absolute values. For example: S_n [MVA], T_1 [s].
4. Function and variable vectors are in lower case, bold face fonts. For example: $\boldsymbol{f}$, $\boldsymbol{g}$, $\boldsymbol{x}$.
5. Jacobian matrices are in lower case, bold face fonts with a sub-index that indicates the variables with respect to which derivatives are computed. For example: $\boldsymbol{g}_{\boldsymbol{y}} = \nabla^T_{\boldsymbol{y}} \boldsymbol{g}$.
6. Hessian matrices are indicated as Jacobian ones but with two sub-indexes. For example: $\boldsymbol{g}_{\boldsymbol{yy}} = \nabla^T_{\boldsymbol{yy}} \boldsymbol{g}$.

7. Other matrices are in upper case, bold face fonts. For example: $\boldsymbol{A}$, $\boldsymbol{I}_n$, $\boldsymbol{W}$.
8. Sets are indicated by calligraphic fonts. For example: $\mathcal{I}$, $\mathcal{N}$.
9. Iterations are indicated with a superscript in brackets. For example: $\boldsymbol{x}^{(i)}$, $\boldsymbol{y}^{(i+1)}$, $\boldsymbol{v}^{(0)}$. The latter indicates the initial guess.
10. The subscript 0 indicates the equilibrium point or the initial value. For example: $\boldsymbol{x}_0 = \boldsymbol{x}(t_0)$.
11. A bar on top of a symbol indicates a phasor. For example: $\bar{v} = ve^{j\theta}$.
12. In case of complex quantities, a superscript asterisk indicates the conjugate. For example: $\bar{v}^* = ve^{-j\theta}$. In case of real quantities, a superscript asterisk indicates the optimal value. For example: p_w^*.
13. A superscript T indicates transpose. For example: $\boldsymbol{A}^T$.
14. Verbatim fonts are used to indicate Python scripts. For example: `list`, `class`. The symbol >>> indicates the Python interactive command line prompt.

Frequent Symbols

The following is only a selection of most common quantities used in the book.

Functions and Equations

$\boldsymbol{f}$ differential equations.
$\boldsymbol{g}$ algebraic equations.
$\boldsymbol{h}$ inequality constraints.
$\mathscr{L}$ Lagrangian function.
$\hat{\boldsymbol{q}}^{(i)}$ vector used for implicit numerical methods.
$\boldsymbol{r}$ reduced gradient.
s differential operator in block diagrams ($s = \frac{d}{dt}$).
ϱ continuation equation.
φ objective function.
$\boldsymbol{\varphi}$ general vector of differential equations.
$\boldsymbol{\psi}$ homotopy map.

Variables and Parameters

a_m converter modulating amplitude.
$\mathscr{E}$ energy.
k_G distributed slack bus variable.
ℓ transmission line length.
m transformer tap ratio.
p_h, $\boldsymbol{p}$ active powers.
q_h, $\boldsymbol{q}$ reactive powers.
$\boldsymbol{s}$ slack variables.
t time.
$\boldsymbol{u}$ discrete variables.

v_h, $\boldsymbol{v}$ bus voltage magnitudes.
$\boldsymbol{w}$ left eigenvector.
$\boldsymbol{x}$ state variables.
$\dot{\boldsymbol{x}}$ first time derivatives of state variables.
$\boldsymbol{y}$ algebraic variables.
$\boldsymbol{z}$ compound vector of variables (e.g., $\boldsymbol{z} = [\boldsymbol{x}^T, \boldsymbol{y}^T]^T$).
α firing angle.
δ_j, $\boldsymbol{\delta}$ generator rotor angles.
$\boldsymbol{\eta}$ controllable variables or parameters.
θ_h, $\boldsymbol{\theta}$ bus voltage angles.
$\varTheta$ temperature.
λ eigenvalue.
μ loading level (continuation parameter).
$\hat{\mu}$ barrier parameter for the interior point method.
$\boldsymbol{\mu}$ independent variables or uncontrollable parameters.
$\boldsymbol{\nu}$ right eigenvector.
$\boldsymbol{\xi}$ general vector of state variables.
$\boldsymbol{\pi}$ dual variables associated with inequality constraints.
$\boldsymbol{\rho}$ dual variables associated with equality constraints.
$\boldsymbol{\tau}$ tangent vector.
τ_e electrical torque.
τ_m mechanical torque.
ϕ transformer phase shift.
ω_j, $\boldsymbol{\omega}$ generator rotor speeds.

Matrices

$\boldsymbol{A}$ generic constant element matrix.
$\boldsymbol{A}_C$ complete system Jacobian matrix.
$\boldsymbol{A}_c^{(i)}$ matrix defined for implicit numerical methods.
$\boldsymbol{A}_S$ state matrix.
$\boldsymbol{B}$ admittance matrix used in dc power flow.
$\boldsymbol{B}'$ and $\boldsymbol{B}''$ admittance matrices used in fast decoupled power flow.
$\boldsymbol{C}$ incidence matrix.
$\boldsymbol{f_x}$ Jacobian matrix $\nabla_{\boldsymbol{x}}^T \boldsymbol{f}$.
$\boldsymbol{f_y}$ Jacobian matrix $\nabla_{\boldsymbol{y}}^T \boldsymbol{f}$.
$\boldsymbol{G}$ nodal conductance matrix.
$\boldsymbol{g_x}$ Jacobian matrix $\nabla_{\boldsymbol{x}}^T \boldsymbol{g}$.
$\boldsymbol{g_y}$ Jacobian matrix $\nabla_{\boldsymbol{y}}^T \boldsymbol{g}$.
$\boldsymbol{N}$ matrix of right eigenvectors.
$\boldsymbol{T}$ connectivity matrix.
$\boldsymbol{W}$ matrix of left eigenvectors.
$\bar{\boldsymbol{Y}}$ admittance matrix.
$\boldsymbol{\varLambda}$ diagonal matrix of eigenvalues.
$\boldsymbol{\varSigma}$ diagonal matrix of singular values.

Constants

A surface area.
B, b_{h0}, b_{k0}, b_{hk} susceptances.
c_p specific heat.
D rotor damping.
f_n nominal frequency in Hz.
G, g_{h0}, g_{k0}, g_{hk} conductances.
H, H_m, H_t inertia constants.
K, K_0, K_i, K_p gains.
ℓ_t total transmission line length.
m_g mass.
R, r_{hk}, r_T resistances.
S_b base power.
T, T_a, T_1 time constants.
V_b base voltage.
X, x_{hk}, x_T reactances.
$\bar{z}$, $\bar{z}_T$ impedances.
$\bar{y}_{hk}$, $\bar{y}_T$ admittances.
$\mathit{\Omega}_b$ nominal synchronous speed in rad/s.
ω_s synchronous speed in pu.

Numbers

n_b number of ac buses.
n_c number of series connections or branches.
n_g number of algebraic functions $\boldsymbol{g}$.
n_G number of generators.
n_u number of discrete variables $\boldsymbol{u}$.
n_x number of state variables $\boldsymbol{x}$.
n_y number of algebraic variables $\boldsymbol{y}$.
n_z number of variables $\boldsymbol{z}$.
n_η number of controllable variables $\boldsymbol{\eta}$.
n_μ number of uncontrollable variables $\boldsymbol{\mu}$.
n_ξ number of general state variables $\boldsymbol{\xi}$.

Sets

$\mathcal{B}$ set of ac buses.
$\mathcal{C}$ set of series connections or branches.
$\mathcal{D}$ set of demands.
$\mathcal{G}$ set of generators.
$\mathcal{N}$ set of dc nodes.
$\mathcal{R}$ set of generator power reserves.

$\mathcal{S}$	set of supplies.
$\mathcal{T}$	set of times.
Ω	set of all devices.
Ω_h	set of devices connectes to bus h.

Device Model Notation

In Part III, the notation is aimed to maintain light expressions. The use of indexes is avoided wherever is possible. Device equations imply the device index i in all variables and parameters. Thus, v_h and θ_h indicate the bus voltage magnitude and phase angle, respectively, while p_h and q_h indicate the active and power injections, respectively. For series devices connected to two buses, the indexes h and k are used.

Bases for Per Unit Values

Throughout the whole book, the bases used for ac values are:

$S_b^{\rm ac}$	three-phase power in MVA.
$V_b^{\rm ac}$	phase-to-phase voltage in kV.

Thus, the current base $I_b^{\rm ac}$ and the impedance base $Z_b^{\rm ac}$ are:

$$I_b^{\rm ac} = \frac{S_b^{\rm ac}}{\sqrt{3} \cdot V_b^{\rm ac}}$$

$$Z_b^{\rm ac} = \frac{V_b^{\rm ac}}{\sqrt{3} \cdot I_b^{\rm ac}} = \frac{(V_b^{\rm ac})^2}{S_b^{\rm ac}}$$

For dc values, the following bases are used:

$S_b^{\rm dc}$	power in MW.
$V_b^{\rm dc}$	voltage in kV.

Thus, the current base $I_b^{\rm dc}$ and the resistance base $R_b^{\rm dc}$ are obtained as follows:

$$I_b^{\rm dc} = \frac{S_b^{\rm dc}}{V_b^{\rm dc}}$$

$$R_b^{\rm dc} = \frac{V_b^{\rm dc}}{I_b^{\rm dc}} = \frac{(V_b^{\rm dc})^2}{S_b^{\rm dc}}$$

For systems where there are both ac and dc devices, it is assumed that $S_b^{\rm dc} = S_b^{\rm ac}$.