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Modelling, Pricing, and Hedging Counterparty Credit Exposure

A Technical Guide

 Springer

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Preface

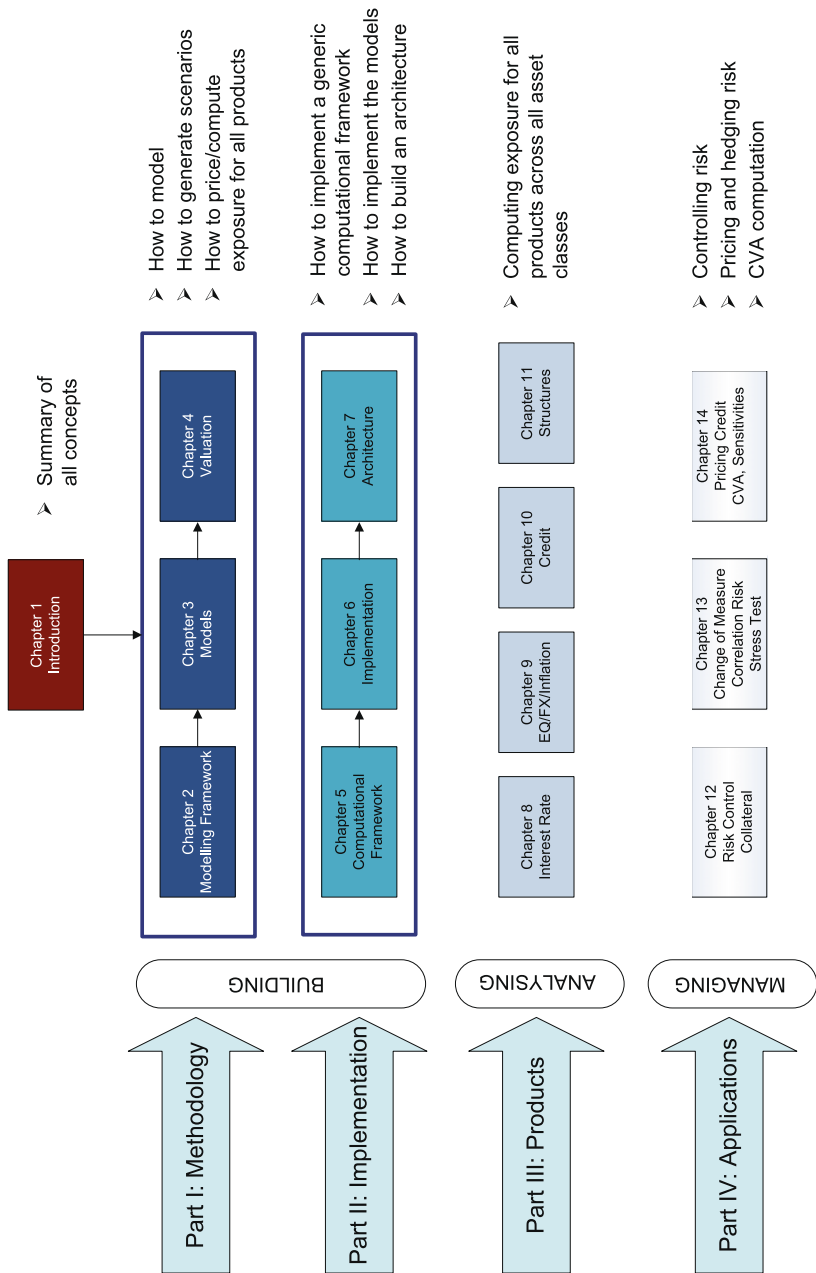
It was the end of 2005 when our employer, a major European Investment Bank, gave our team the mandate to compute in an accurate way the counterparty credit exposure arising from exotic derivatives traded by the firm. As often happens, exposure of products such as, for example, exotic interest-rate, or credit derivatives were modelled under conservative assumptions and credit officers were struggling to assess the real risk. We started with a few models written on spreadsheets, tailored to very specific instruments, and soon it became clear that a more systematic approach was needed. So we wrote some tools that could be used for some classes of relatively simple products. A couple of years later we are now in the process of building a system that will be used to trade and hedge counterparty credit exposure in an accurate way, for all types of derivative products in all asset classes. We had to overcome problems ranging from modelling in a consistent manner different products booked in different systems and building the appropriate architecture that would allow the computation and pricing of credit exposure for all types of products, to finding the appropriate management structure across Business, Risk, and IT divisions of the firm.

In this book we describe some of our experience in modelling counterparty credit exposure, computing credit valuation adjustments, determining appropriate hedges, and building a reliable system.

What do we mean by all of this? Counterparty credit exposure is the amount a company could potentially lose in the event of one of its counterparties defaulting. At a general level, computing credit exposure entails simulating in different scenarios and at different times in the future, prices of transactions, and then using one of several statistical quantities to characterise the price distributions that has been generated. Typical statistics used in practice are (i) the mean, (ii) a high-level quantile such as the 97.5% or 99%, usually called *Potential Future Exposure* (PFE), and (iii) the mean of the positive part of the distribution, usually referred to as *Expected Positive Exposure* (EPE).

With these measures and default probability information or counterparty CDS premia, it is then possible to price counterparty risk. In the financial industry, the economic value of this risk is generally called *Credit Valuation Adjustment* (CVA).

Reader's Roadmap



As we will have occasion to see later in this book, it can be computed as the price of a Credit Default Swap paying the Expected Positive Exposure. Equivalently expressed, CVA is the price of a new type of hybrid product, the so-called *Contingent Credit Default Swap* (C-CDS), which pays the value of the exposure (floored at zero) upon default of the counterparty.

The credit crisis which started in 2007 clearly shows why it is of crucial importance for any company entering a derivative business to (i) measure counterparty exposure, (ii) compute capital requirements, and (iii) hedge counterparty risk. Measuring counterparty exposure is important for setting limits on the amount of business a firm is prepared to do with a given counterparty; hedging it gives a possibility of mitigating it and transferring risk; and from a regulatory perspective there is significant pressure on financial institutions to have the capability of producing accurate risk measures to compute capital. In addition, computing counterparty exposure can also give insights into prices of complex products in potential future scenarios. It seems that what was until recently a Risk Control function attracting relatively limited attention, is now becoming a central activity of all major financial institutions, requiring significant resources from all parties.

Our approach to counterparty credit exposure analysis is quantitative. The focus is on mathematical modelling, simulation techniques using various Monte Carlo approaches, and pricing. In contrast, we are only marginally interested in assessing the quality of counterparties or in analysing historical market data in order possibly to forecast future behaviours of the economy, or in risk and regulatory aspects of the problem. We consider derivative products and complex structures which are usually traded in Investment Banks, and focus on practical aspects of the problems at hand. All models used in our analyses are tested with practical data and real transactions. Given this quantitative focus, we sometimes refer to our work as Credit Quantification.

The book is divided into four parts, (I) Methodology, (II) Architecture and Implementation, (III) Products, and (IV) Hedging and Managing Counterparty Risk. In Part I we present a generic simulation framework, which can be used to compute counterparty exposure for both vanilla and exotic products. We show how the classical Monte Carlo framework, where price distributions are computed by generating thousands of scenarios and by explicitly pricing the product at each point in time and at each scenario, is a special case of our more general framework. The classical Monte Carlo approach works well only for products that can be priced in analytical or quasi-analytical form. It is not practical for products that cannot be priced in closed form and require, for instance, a Monte Carlo or lattice pricing approach. Typical examples are products with callability features or exotic interest-rate transactions. We show how in these cases American Monte Carlo techniques used generally for pricing can also be applied efficiently to compute exposure, as they provide intermediate valuations over time and scenarios.

Part II shows how our simulation framework naturally leads to the implementation of a software architecture and the definition of a programming language that allows the computation of both vanilla and exotic products in a scenario-consistent way. In practice, in a large financial institution one of the main problems in building

counterparty exposure systems, is to integrate different products, booked in different systems and priced using libraries written in different languages and with different technologies, in order to compute portfolio exposure across different businesses. We show that our approach leads to an architecture that can integrate other systems in a natural way.

In Part III we consider how to compute exposure for different products. We show how the general techniques and models described in Part I and the architecture described in Part II can be used in practice.

Finally in Part IV things are put together. We consider how to perform risk management and risk control of counterparty exposure on a portfolio basis. We describe different aggregation techniques and a standard set-up that uses collateral to mitigate exposure. We also analyse how to model wrong-way/right-way exposure, where transaction price fluctuations and quality of the counterparty are correlated and we address the problem of changing the reference probability measure after the simulation has been performed. The final chapter is dedicated to pricing counterparty credit exposure and to computing CVA and CVA sensitivities not only to credit spread, but also to market risk factors. The whole book can be seen as a roadmap to achieve this goal.

One note to conclude: in our work we describe and use well-established simulation and pricing techniques. Our goal is not to suggest new or more sophisticated algorithms. It is rather to show how well-known algorithms can be put together and used to compute counterparty credit exposure and which limitations have to be taken into consideration if we want to move from vanilla products to complex exotic transactions.

London, September 2009

Giovanni Cesari
John Aquilina
Niels Charpillon
Zlatko Filipović
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