Generalized Gaussian Error Calculus

Michael Grabe

Generalized Gaussian Error Calculus

With 47 Figures

Springer

Dr. rer. nat. Michael Grabe
Am Hasselteich 5
38104 Braunschweig, Germany
michael.grabe@gmx.net

ISBN 978-3-642-03304-9 e-ISBN 978-3-642-03305-6
DOI 10.1007/978-3-642-03305-6
Springer Heidelberg Dordrecht London New York

Library of Congress Control Number: 2009940174

Cover design: eStudio Calamar Steinen

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

To Lucy, Niklas, Finley, and Rafael

Preface

The book of nature is written in the language of mathematics
Galileo Galilei, 1623

Metrology strives to supervise the flow of the measurand's true values through consecutive, arbitrarily interlocking series of measurements. To highlight this feature the term *traceability* has been coined. Traceability is said to be achieved, given the true values of each of the physical quantities entering and leaving the measurement are localized by specified measurement uncertainties.

The classical Gaussian error calculus is known to be confined to the treatment of random errors. Hence, there is no distinction between the true value of a measurand on the one side and the expectation of the respective estimator on the other. This became apparent not until metrologists considered the effect of so-called unknown systematic errors. Unknown systematic errors are time-constant quantities unknown with respect to magnitude and sign. While random errors are treated via distribution densities, unknown systematic errors can only be assessed via intervals of estimated lengths.

Unknown systematic errors were, in fact, addressed and discussed by Gauss himself. Gauss, however, argued that it were up to the experimenter to eliminate their causes and free the measured values from their influence. Unfortunately, this is not possible. Considering the present state of measurement technique, unknown systematic errors are of an order of magnitude comparable to that of random errors and this causes the Gaussian error calculus to break down. Consequently, the metrological community needs to consider how the error calculus to-be should address the coexistence of random errors and unknown systematic errors.

In the late 1970s, a seminar entitled *On the Statement of the Measuring Uncertainty* [14] was held at the Physikalisch-Technische Bundesanstalt in Braunschweig which, regrettably enough, induced a bifurcation of error calculus: one branch attempted to save the Gaussian approach by formally randomizing unknown systematic errors and thus producing the *Guide to the*

Expression of Uncertainty in Measurement, GUM for short [15]; the other proposed a revision from scratch and issued an essentially new, generalized version of the Gaussian error calculus. This latter approach will be discussed here.

The proceeding considers time-constant unknown systematic errors to spawn biased estimators thus preventing the true values of the measurands and the expectations of the respective estimators from coinciding. Eventually, this physically founded distinction gave birth to the term traceability.

Independently of the question of how to treat unknown systematic errors, the author devotes attention to another point of interest relating to the treatment of random errors. Commonly, random errors are considered normally distributed, at least approximately, and we shall, as a matter of course, retain this assumption. The inclusion of the multidimensional model in error calculus, an obvious extension, seems as yet to have been overlooked. For this model to be deployed properly, each of the variables involved is required to hold the same number of repeated measurements. This apparently banal request turns out exceedingly beneficial: it solves the cumbersome problem error calculus traditionally suffers from when it comes to assign confidence intervals in error propagation. Renouncing the stipulation of equal numbers of repeated measurements puts experimenters beyond the validity of the distribution density of the empirical moments of second order—and this very observation causes the trouble.

The generalized Gaussian approach presented here produces reliable, easy to attain measurement uncertainties meeting the demands of traceability. At the same time, the approach features the properties of a building kit: any overall uncertainty is the sum of the contribution of random errors taken from a confidence interval as provided by Student and the contribution of unknown systematic errors, as expressed by an appropriate worst case estimation.

The book supplements the author's monograph*Measurement Uncertainties in Science and Technology* [28]. Its principles have, however, been tautened and cast into an order attaching prior-ranking importance to metrology's starting point, the traceability of physical units, physical constants and physical quantities at large. In addition, the previously discussed formalism has been extended in some respects. Finally, the procedures are now illustrated by scores of numerically based diagrams helping the reader to pursue the idea of traceability.

Braunschweig, November 2009 *Michael Grabe*

Contents

Part III Error Propagation

Part IV Essence of Metrology

Part V Fitting of Straight Lines

Part VI Fitting of Planes

Part VIII Non-linear Fitting

Part IX Appendices