



# Adaptation, Learning, and Optimization    Volume 1

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Jingqiao Zhang and Arthur C. Sanderson

# Adaptive Differential Evolution

A Robust Approach to Multimodal Problem  
Optimization

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# Foreword

I first met Jingqiao when he had just commenced his PhD research in evolutionary algorithms with Arthur Sanderson at Rensselaer. Jingqiao's goals then were the investigation and development of a novel class of self-adaptive differential evolution algorithms, later called JADE. I had remarked to Jingqiao then that Arthur always appreciated strong theoretical foundations in his research, so Jingqiao's prior mathematically rigorous work in communications systems would be very useful experience. Later in 2007, when Jingqiao had completed most of the theoretical and initial experimental work on JADE, I invited him to spend a year at GE Global Research where he applied his developments to several interesting and important real-world problems.

Most evolutionary algorithm conferences usually have their share of innovative algorithm oriented papers which seek to best the state of the art algorithms. The best algorithms of a time-frame create a foundation for a new generation of innovative algorithms, and so on, fostering a meta-evolutionary search for superior evolutionary algorithms. In the past two decades, during which interest and research in evolutionary algorithms have grown worldwide by leaps and bounds, engaging the curiosity of researchers and practitioners from many diverse science and technology communities, developing stand-out algorithms is getting progressively harder.

JADE stands out as a unique algorithm for these principal reasons: it works very well, it has an elegant theoretical foundation, it has been recognized as the winner in the 2008 IEEE World Congress on Computational Intelligence competition, and it is very user-friendly relieving the user of the task of algorithm parameter setting. As most decision and control problems may be cast in the semantics of an optimization or search problem, powerful new methods that can efficiently handle nonlinear, discontinuous, and high-dimensional spaces have a very significant real-world impact.

The No Free Lunch Theorem for optimization states that for any optimization algorithm, an elevated performance over one class of problems is offset by degraded performance over another class. Thus, the performance average

of a given optimization algorithm over the entire class of potential problems is constant. If an algorithm performs better than random search for some problems, it will perform worse than random search for other problems, maintaining a constant performance average. These results suggest that any one set of potentially optimal algorithms can be considered so only for a limited subset of problems and their use cannot result in consistently superior performance over the entire space of optimization problems, as was previously generally expected. However, an algorithm can outperform another if it is specialized to the structure of the specific problem class under consideration. An alternative approach, as developed in JADE, is to embed self-adaptation in the search strategy that can automatically adapt control parameters in a manner responsive to a wide range of search space characteristics and at different stages of the evolutionary search. This need for adaptively evolving algorithm parameters as the evolutionary search proceeds is highlighted by the theoretical results underpinning JADE. Further, the self-adaptation characteristic of JADE enhances its usability as the user does not need to have significant knowledge of the interaction between control parameters and algorithm behavior.

The book's authors present an excellent balance of rigor, clarity, and real-world focus. This book, for the technical reader, is an exciting document covering a powerful class of state of the art self-adaptive differential evolution algorithms, JADE, that has demonstrated its mettle not only on several standard experimental test spaces, but also importantly on several complex real-world optimization problems spanning combinatorial auctions, financial credit decision-making, and flight route planning in air-traffic control systems.

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Finally, on a personal note, the first author would like to express his sincerest thank and gratitude to his advisor Dr. Arthur C. Sanderson and his internship supervisor Dr. Raj Subbu for their continuous guidance, inspiration and encouragement, without which this work would not have been possible.

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# Acronyms

AEC	Adaptive Evolution Control
CA	Combinatorial Auction
CC	Cooperative Coevolution
DE	Differential Evolution
EA	Evolutionary Algorithms
EP	Evolutionary Programming
ES	Evolution Strategy
ETPM	Empirical Transition Probability Matrix
FESS	Function Evaluations in Successful Runs
GA	Genetic Algorithm
GP	Genetic Programming
MOEA	Multi-objective Evolutionary Algorithm
PSO	Particle Swarm Optimization
RBF	Radial Basis Function
SNR	Signal-to-noise ratio
SR	Success Rate
SP	Success Performance
TPM	Transition Probability Matrix