Jacek Kluska

Analytical Methods in Fuzzy Modeling and Control

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Foreword

The technologies of fuzzy systems and fuzzy controllers, in particular, have been applied with great success to numerous real world applications. The number of entries in the INSPEC database with keywords "fuzzy modeling" and "fuzzy control" dated 1969–2007 is 1,541 and 9,728, respectively. In spite of the evident progress reported in terms of concepts, algorithmic developments and engineering practice, there are still a number of challenging and highly relevant problems. Unfortunately, the existing publications are rather silent when it comes to reporting comprehensive solutions to them. The two challenges become particularly apparent and have been triggered by the growing complexity of the applications. The first evident challenge we are faced with is the curse of dimensionality. Rule–based systems and fuzzy rule–based systems are quite affected by this phenomenon especially when tackling problems of high dimensionality. The second one concerns a way of constructing fuzzy models which are accurate yet highly interpretable.

The author of the monograph has focused on these two vital problems and offered an interesting, original and practically relevant insight into their solutions. When dealing with fuzzy modeling, the book focuses on a broad class of Takagi–Sugeno–Kang (TS) fuzzy models – a highly legitimate choice given a wealth of literature on these constructs and a great deal of their applications. Furthermore the TS fuzzy models have been a subject of numerous analytical studies which have resulted in a series of interesting findings. This situation stands in a deep contrast with the most studies carried out in the realm of fuzzy control where analytical methods are not very common.

The analytical methods are beneficial to the better understanding of the advantages of the technology of fuzzy systems and its usage to the fullest extent when dealing with real–world problems. The book authored by Jacek Kluska is an important endeavor along this timely line of the development of fuzzy systems. While the author relates to an interesting treatise authored by Hao Ying (*Fuzzy control and modeling. Analytical foundations and applications.* IEEE Press, New York 2000), the book brings new and very much attractive ideas and presents important findings. The author not only re–visited and cast some Ying's results in an original fashion but further developed the Takagi–Sugeno fuzzy systems endowed with polynomial membership functions.

There are new notions and interesting results. The author introduced the notions of the generator and the fundamental matrix of the rule–based system and offered a convenient matrix description of the multiple–input multiple–output fuzzy system. Next, provided was a clear mathematical relation between the system of fuzzy rules and the systems described by "classical" differential or difference equations. The new and important are recurrence theorems dealing with rule–based systems with generalized classes of membership functions. It has been shown that those functions play an essential role in battling the ubiquitous curse of dimensionality.

Through a series of theorems the author established a one–to–one correspondence between the fuzzy systems and their classical counterparts and provided a detailed solution to many practical problems of substantial dimensionality.

Numerous examples covered in the text demonstrate the usefulness of the analytical methods of the fuzzy modeling in application to physical systems. The book builds a bridge between the highly interpretable fuzzy rule–based systems, classical control methods based on Boolean logic, multivalued logic and the conventional control theory, including its classic constructs of PID controllers.

Owing to the analytical approach the author developed an algebraic theory of rule–based systems, worked out an effective identification algorithms for a certain class of nonlinear dynamical systems, and proposed an interesting new classification system involving a collection of highly interpretable fuzzy rules.

A truly outstanding feature of this book is a mathematical rigor with which the author treats the subject matter and presents the reader with carefully structured ideas and algorithmic pursuits. All in all, the book can be highly recommended to researchers and practitioners interested in exploiting analytical methods of fuzzy modeling and control, system identification and diagnostics. Definitely this well–timed volume is a testimony to the rapid progress and a significant wealth of concepts and applications of Computational Intelligence.

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Preface

This book does not contain an elementary mathematics of fuzzy systems such as fuzzy sets, operations on fuzzy sets, Boolean logic, triangular norms (t-norms), t-conorms, implications, fuzzy relations, fuzzy reasoning methods, the fuzzy controller architecture, the Mamdani type fuzzy controller, etc., because of the flood of papers and books on these topics. It is assumed that the reader is familiar with the fundamentals of the fuzzy modeling and with the foundations of Boolean logic and conventional control methods, including PID control.

This book is focused on mathematical analysis and rigorous design methods for fuzzy control systems based on Takagi-Sugeno fuzzy models, sometimes called Takagi-Sugeno-Kang models. We present a rather general analytical theory of exact fuzzy modeling and control of continuous and discrete-time dynamical systems. The main attention is paid to usability of the results for the control and computer engineering community and therefore simple and easy for linguistic interpretation knowledge-bases have been used. The approach is based on the author's theorems concerning equivalence between widely used Takagi-Sugeno systems and some class of multivariate polynomials. It combines the advantages of fuzzy system theory and classical control theory. Classical control theory can be applied to modeling of dynamical plants and the controllers. They are all equivalent to the set of Takagi-Sugeno type fuzzy rules. The approach combines the best of fuzzy and conventional control theory. It enables linguistic interpretability (also called transparency) of both the plant model and the controller. In the case of linear systems and some class of nonlinear systems, the engineer can in many cases directly apply well-known classical tools from the control theory both for analysis, and the design of the closed-loop fuzzy control systems.

The main objective of this book is to establish comprehensive and unified analytical foundations for fuzzy modeling using Takagi-Sugeno rule scheme and their applications for fuzzy control, identification of some class of nonlinear dynamical processes and classification problem solver design. After an excellent book of Ying [207], this is probably the second book which attempts to rigorously show that the fuzzy control is not a collection of applications without a solid theory. We pay special attention to the use of precise language to introduce the definitions and concepts, and to prove the conclusions.

Intended Readership of the Book

This self-contained textbook is intended for anyone who is interested in analytical aspects of fuzzy modeling and control applying the widely used Takagi-Sugeno rule scheme and wants to know precisely their connections with the classical counterparts. It is a self-study book for engineering professionals in diverse technical fields and industries, especially those in the fields of control and computer science. It aims at an audience of graduate and Ph.D. students as well. We assume that the reader has elementary background corresponding to an introductory course in automatic control, linear algebra and fundamentals of switching theory and logic design.

After reading Chapters 2 and 4, it can be studied in many ways, according to the particular interests of the reader. The book can be used together with the books on fundamentals of the control theory, artificial neural networks and other methods on machine learning. If a practicing engineer wants to apply the results of this book quickly, then the proofs of the lemmas and theorems may be skipped.

Originality of the Book

This book is focused on the rigorous mathematical methods of fuzzy modeling and control systems design based on the widely used Takagi-Sugeno rule scheme (TS for short), but it is not intended as a collection of existing results on fuzzy systems or fuzzy control. We present a new analytical theory of exact fuzzy modeling of continuous and discrete-time dynamical systems and logic systems which can be applied to solve the control, identification and classification problems encountered in practice. Therefore rather simple and highly interpretable knowledge-bases are used, putting a particular emphasis on the matrix calculus, symbolic calculus and recurrence. The approach is based on the author's theorems concerning equivalence between the Takagi-Sugeno systems and some class of multivariate polynomials, which combines the advantages of fuzzy system theory and classical control theory. Among others, it enables linguistic interpretability of the plant models and the controllers. Using the results developed in this book, the engineer can in many cases directly apply well-known tools from the conventional control theory (e.g. PID control) or binary logic design theory (e.g. combinational or sequential circuits), both to the analysis and design of the linear and some class of nonlinear closed-loop fuzzy control systems.

Several notions and results are new in this book, which are unavailable in any other book. To them belong the notions of the generator and the fundamental matrix of the TS system, the matrix description of the multiple-input-multiple-output (MIMO) TS system and new results on recurrence for the rule-based systems involving the first- and the second-order polynomials as the membership functions of fuzzy sets defined for the input variables. The book contains the proofs of the results in order to maintain a rigorous approach. Many examples included in the text illustrate usefulness of the analytical methods of the fuzzy modeling to many physical systems. The results obtained in this book are compared with other ones to show the advantages of the proposed procedures.

The material contained in this book is oriented towards the algorithms that are practically useful. We use analytical and systematic approach to the synthesis and analysis of the models. Thanks to this, a comparison of the methods developed in this book with the methods obtained by other authors is straightforward. Symbolic quantities are mainly used to ensure the generality of outcomes. Seldom, if ever, will numerical data be taken, to increase transparency of the examples. The book contains many examples concerning exact fuzzy modeling and control of real systems. We show theoretically and by examples that the fuzzy rule-based systems with the linear membership functions deserve a special attention not only from the theoretical point of view, but also they should be attractive for practitioners. The analytical results reinforce our belief that many successful applications of the fuzzy control cannot be a matter of chance.

Overview of the Book

The book consists of seven chapters. Chapter 2 provides the notion of the generator and the fundamental matrix of the rule-based system which are crucial for the book. One of the theorems establishes an exact relationship between the collection of fuzzy rules and a class of functions to which they are equivalent. It plays a crucial role in the modeling of many physical systems by using highly interpretable fuzzy rules. We show that the considered fuzzy rule-based system is nothing else but a part of the well-known Kolmogorov-Gabor polynomial. We prove that by formulating the consequents of the fuzzy rules which should express a given function, the only information needed by an expert is the values of this function in all vertices of the hypercuboid. In this chapter we introduce a compact matrix description of the set of fuzzy rules. The rule-based systems which use the linear membership functions of fuzzy sets for input variables are called the P1-TS systems. They are highly interpretable and therefore they are important from the engineering point of view. Finally, we consider an equivalence problem of the rule-bases in the context of the matarules taking into account that in reality the rule-bases can be noncomplete and/or contradictory ones.

Motivated by the "curse of dimensionality problem" of the fuzzy rule-based systems, Chapter 3 provides several results to make a calculation of the crisp system output feasible. We give some features of the fundamental matrix and its inverse. Thanks to one of theorems, the fundamental matrix inverse can be found recursively using multiplication operations only, instead of using classical inversion procedures. One of the main advantages of this chapter is providing a recursive procedure to solve the problem of "How to obtain the function performed by the rule-based system containing a large number of rules". To the best of the author's knowledge this problem has not been solved in the literature as yet. We show that thanks to the recursion, the curse of dimensionality problem can be substantially reduced. The computational architecture of the recursion can be viewed as a feedforward neural network. As an example of application of the recursion, the rule-based system with 6 inputs was considered. However, it is not a big problem to consider a P1-TS system with about 10 inputs. Next we show that the P1-TS systems can be used for the exact modeling of the nonlinear continuous or discrete-time dynamical systems, where the inputs of the fuzzy rule-based system are more abstract quantities and the outputs refer to the system structure. Such approach coincides in many respects with the one described in [184, where the system inputs can contain known premise variables that are not functions of the control input, but they may be functions of the state variables, external disturbances and/or time. For every input variable we assume two complementary membership functions that cannot be monotonic or linear. The advantages of our approach are exemplified. For the inverted pendulum system we obtain a better result than in other works. By using recurrence we can easily check validity of other models of nonlinear systems in the P1-TS form, e.g. a translational oscillator with an eccentric rotational proof mass actuator, a vehicle with triple trailers and many other dynamical systems discussed in the literature. In this chapter we show that application of the Taylor series expansion can be very attractive in practice. In one example we use 4th-degree Taylor polynomials for a good approximation of nonlinear functions at the equilibrium point of the dynamical system. The result is much better than the one obtained by the linearization of differential equations around the equilibrium. By using the Taylor series expansion we obtain a small number of highly interpretable fuzzy rules. Finally, we give the best evaluation for the lower and upper bound of the function, to which the rule-based P1-TS system is equivalent.

In order to obtain a richer class of functions to which the fuzzy rule-based system is equivalent, in Chapter 4 we use polynomials of the degree higher than one, as the membership functions of fuzzy sets. A special attention is paid to the TS systems which use the second degree polynomials. We show that it is not possible to obtain any second degree polynomial function, to which a TS rule-based system is equivalent, on the assumption that only two complementary membership functions as the second degree polynomials are defined for the input variables. However, three quadratic membership functions suffice to model every second degree polynomial function. For the zero-order TS system, we define for every input variable the set of three highly interpretable normalized membership functions as the second degree polynomials. The TS systems that use such fuzzy sets we call P2-TS systems. Such systems are thoroughly investigated. One of theorems says that the crisp output of the MISO P2-TS system in the vertex of the hypercuboid is exactly the same as the appropriate conclusion of the fuzzy rule contained in the rule-base. For the P2-TS systems both the generator and the fundamental matrix are defined. The fundamental matrix and its inverse are very important for the considered systems, since they enable one to establish an exact relationship between the consequents of the "If-then" rules and the parameters that define the crisp function, to which the rule-based system is equivalent. Therefore the procedures of how to compute the fundamental matrix and its inverse are given. The examples show that P2-TS systems have highly interpretable rule-bases when we use individual fuzzy rules or the metarules. The curse of dimensionality problem is much more serious for the P2-TS systems than the one for the P1-TS systems. Therefore, we develope the recursive procedures for the computation of both the inverse of the fundamental matrix and the crisp output of the P2-TS systems. The theorems say that we do not need to inverse large matrices to obtain the crisp output of the P2-TS systems. As a result of these theorems, the curse of dimensionality in P2-TS systems is substantially weakened. The results of this chapter can be easily generalized for the MIMO case. After this chapter we are able to thoroughly generalize the results for the TS systems with the membership functions that are polynomials of the degree $d \geq 3$. However, we should realize that the number of complete and noncontradictory rules will rapidly grow and the analysis will become more and more complicated. Both P1- and P2-TS systems are able to model a large class of real nonlinear processes. Therefore, if it is not necessary, we should not complicate our models in the engineering practice.

Chapter 5 mainly focuses on the P1-TS systems as the simplest and the most transparent among fuzzy rule-based systems with polynomial membership functions. In order to show that there are quite a lot of applications of P1-TS systems, many examples of exact modeling of conventional systems are given, especially in relation to nonlinear dynamical processes modeling and control. The P1-TS systems with two and more inputs are comprehensively investigated in the subsequent sections of Chapter 5, considering interpretability issue. It is exemplified that by using a multi-valued logic for highly nonlinear dynamical process, one can design an acceptable control algorithm expressed by the P1-TS system fuzzy rules. We show a connection between P1-TS systems and classical combinational logic systems. The fuzzy rule-based systems with inputs and outputs from the unity intervals are discussed in the context of generalized operators such as triangular norms, t-conorms, implications, etc. In this way, an unavoidable connection between fuzzy rule-based systems and Boolean algebra becomes apparent. We exemplify that the theory of P1-TS systems can be used to transform some control algorithms, formerly obtained with the use of Boolean logic, into the fuzzy domain. The highly interpretable rule-bases are constructed for the systems with three and more inputs not only for abstract processes, but also for real dynamical plants, e.g. a NARX model, fuzzy J-K flip-flop, Euler equations for a rigid body, Chen's attractor, the human immunodeficiency virus, magnetic suspension system, low order atmospheric circulation process and induction motor. The theory of P1-TS systems is also used for optimal analytical design of the well-known PID controller, working in the closed-loop control system for some class of the linear and nonlinear second order plants. Such a controller in the form of P1-TS system is optimal with respect to typical requirements for automatic control systems. After studying analytical results it is clear why the fuzzy PID controller as a P1-TS system can be better than the conventional PID algorithm. Next, we show that using our systematic approach, the so called "controller with variable gains" introduced by Ying [205], [206] can be easily obtained. In the last sections of Chapter 5 exact modeling of single input dynamical systems is investigated. Similarly as in the preceding sections we assume that nonlinear dynamical system is a collection of linear dynamical subprocesses. However, in contrast to the previous approach, where the inference was concerned with the structure parameters represented by matrices describing local linear models, the nonlinear model of the whole system is now inferred according to the original Takagi-Sugeno inference method. Based on this inference, we identify the class of dynamical systems to which the rule-based system is equivalent. Theoretical results are exemplified by exact fuzzy modeling of the van de Vusse reaction and Rössler chaotic system. Next we describe the architecture of the P1-TS system as the fuzzy model of conventional MIMO linear dynamical system. In Section 5.8 we show that the idea of TS systems with two linear membership functions of fuzzy sets can be easily extended to the systems with triangular fuzzy partition. The triangular membership functions can be substituted by other nonlinear membership functions which have the same support and the same monotonicity intervals. As a practical example of using the systems with triangular fuzzy partition, we present a sensor-based navigation system for a mobile robot. Chapter 5 ends with supplementary results for P1-TS systems. The outcomes concern the necessary and sufficient condition of linearity for such rule-based systems, the first-order P1-TS systems and the zero-order systems with contradictory rule-base. In the last section we show that the system without contradictions is a special case of the rule-based system with contradictions. For such systems we introduce a generalized fundamental matrix of the P1-TS, which can be easily extended to the P2-TS systems.

In Chapter 6 we investigate the identification problem of multilinear dynamical systems from observation data. Based on analytical results concerning exact fuzzy modeling of multilinear dynamical systems which provide necessary and sufficient conditions for transformation of fuzzy rules into crisp model, we prove the theorem on existence of the solution in the form of the P1-TS system. To get a solution we propose to use a batch procedure or recursive least squares method. The methodology preserves the interpretability of the fuzzy models, which is a key property of the considered rule-based systems and can be applied to continuous or discrete-time multilinear systems.

The proposed computation method can be viewed as a supervised learning algorithm for the adaptive linear neural network.

Chapter 7 provides a method for obtaining a set of highly interpretable "If-then" rules for the P1-TS system as an optimal (in the sense of a good generalization ability) binary classification problem solver. We use the results developed in the previous sections, especially related to modeling of the rule-based system from the input-output data, and the contradictory rules. The idea of constructing the classifier involves the theory of generators, fundamental matrices and support vector machines.

The bibliography at the end of the book lists the publications cited in the text as well as other relevant items that are not cited. Given a vast amount of papers and books, it is inevitable that the bibliography is still incomplete.

Of course, it is impossible to cover the entire spectrum of topic areas in one volume. A connection between the highly interpretable fuzzy "If-then" rules and some methods of artificial intelligence such as neural networks or kernel-based methods, was only signalled in this volume. Many of the results contained in this book establish a good starting point for stability and robustness analysis of fuzzy control systems and developing new learning and adaptation tools for intelligent control and diagnostic systems, which could be included in the future edition.

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