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# Theoretical Molecular Biophysics

With 178 Figures

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## Preface

Biophysics deals with biological systems, such as proteins, which fulfill a variety of functions in establishing living systems. While the biologist uses mostly a phenomenological description, the physicist tries to find the general concepts to classify the materials and dynamics which underly specific processes. The phenomena span a wide range, from elementary processes, which can be induced by light excitation of a molecule, to communication of living systems. Thus, different methods are appropriate to describe these phenomena. From the point of view of the physicist, this may be Continuum Mechanics to deal with membranes, Hydrodynamics to deal with transport through vessels, Bioinformatics to describe evolution, Electrostatics to deal with aspects of binding, Statistical Mechanics to account for temperature and to learn about the role of the entropy, and last but not least Quantum Mechanics to understand the electronic structure of the molecular systems involved. As can be seen from the title, Molecular Biophysics, this book will focus on systems for which sufficient information on the molecular level is available. Compared to crystallized standard materials studied in solid-state physics, the biological systems are characterized by very big unit cells containing proteins with thousands of atoms. In addition, there is always a certain amount of disorder, so that the systems can be classified as complex. Surprisingly, the functions like a photocycle or the folding of a protein are highly reproducible, indicating a paradox situation in relation to the concept of maximum entropy production. It may seem that a proper selection in view of the large diversity of phenomena is difficult, but exactly this is also the challenge taken up within this book. We try to provide basic concepts, applicable to biological systems or soft matter in general. These include entropic forces, phase separation, cooperativity and transport in complex systems, like molecular motors. We also provide a detailed description for the understanding of elementary processes like electron, proton, and energy transfer and show how nature is making use of them for instance in photosynthesis. Prerequisites for the reader are a basic understanding in the fields of Mechanics, Electrostatics, Quantum Mechanics, and Statistics. This means the book is for graduate students, who want to

specialize in the field of Biophysics. As we try to derive all equations in detail, the book may also be useful to physicists or chemists who are interested in applications of Statistical Mechanics or Quantum Chemistry to biological systems. This book is the outcome of a course presented by the authors as a basic element of the newly established graduation branch “Biophysics” in the Physics Department of the Technische Universität Muenchen.

The authors thank Dr. Florian Dufey and Dr. Robert Raupp-Kossmann for their contributions during the early stages of the evolving manuscript.

Garching,  
May 2010

*Sighart Fischer*  
*Philipp Scherer*

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