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André Unterberger

Alternative Pseudodifferential Analysis

With an Application to Modular Forms

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Author

André Unterberger

Mathématiques
Université de Reims
Moulin de la Housse, BP 1039
51687 Reims Cedex 2
France
andre.unterberger@univ-reims.fr

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To François Treves

Preface

The subject of the present work is pseudodifferential analysis: the motivations lie in harmonic analysis and modular form theory. So far as the last two domains are concerned, nothing more than some minimal familiarity is needed: some knowledge of the metaplectic representation, and of the definition of holomorphic and nonholomorphic modular forms, will help. Even though the symbolic calculus introduced here is entirely new, and does not depend on any technical result concerning pseudodifferential operators, it would not be honest to claim that no previous acquaintance with that field is necessary: the analysis developed here is strikingly different from the usual one, some knowledge of which – in particular, its representation-theoretic aspects – is needed for comparison.

Modular form theory is a very appealing subject: some time ago already, we tried to approach it from an angle which, to us, was much more familiar, that of pseudodifferential analysis. It is possible to realize nonholomorphic modular forms as distributions in the plane [35, Sect. 18], the main benefit being that they can then be considered as symbols for a calculus of the usual species, to wit the Weyl calculus. Yes, there are difficulties on the way toward developing the symbolic calculus of associated operators, since distributions on $\mathbb{R}^2$ which correspond to modular forms, though beautiful objects from the point of view of arithmetic, are extremely singular. Still, one can survive these difficulties, as shown in [36].

Only the nonholomorphic modular form theory could be reached in this way. Needless to say, we tried to incorporate holomorphic modular form theory as well: this cannot work to a full extent, and the best one can do in this direction will be summed up in Sect. 5.2 of the present work. Then, in an independent piece of work [38], partly motivated by Physics, we introduced the "new" anaplectic analysis – like many new things, it is only a coherent rearrangement of old ones – and it turned out, to our unanticipated satisfaction, that this solved our old problem.

Only one-dimensional anaplectic analysis will concern us here – the higher-dimensional case is considerably harder – and, of course, we are not assuming that the reader has read, or borrowed, our book on the subject. It is our opinion that the version presented here, in Sects. 2.2 and 4.1, in which no proofs are given, will make easy reading. Though our main current interest in anaplectic analysis lies with Physics, it is clear, to us, that the approach to holomorphic modular form theory it leads to deserves to be explored further.

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