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For Mira

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## Preface

Ernest Rutherford (New Zealand–British physicist, 1871–1937), the 1908 Nobel Laureate who discovered the existence of atomic nuclei, is famously quoted as having said: “Physics is the only real science. All the rest is butterfly collecting.” Or something to that effect. I like to include this quote in my introductory remarks at the first class meetings of the physics courses I teach.

I have seen that there are those who interpret this as a put-down of amateurs (butterfly collectors) in science. However, my own interpretation of Rutherford’s statement is that he is claiming that, except for physics, all of the rest of science is involved merely in collecting facts and classifying them (butterfly collecting). It is physics, unique among the sciences, that is attempting to find *explanations* for the classified data.

The periodic table of the chemical elements, originally proposed by Dmitri Ivanovich Mendeleev (Russian chemist, 1834–1907), presents an example of this. Chemists toiled to discover the chemical elements and their properties and then classified the elements in the scheme that is expressed by the periodic table. Here was the chemists’ butterfly collecting. It took physicists to *explain* the periodic table by means of quantum theory.

Rutherford’s assessment of science might well have held a large degree of validity in the 19th and early 20th centuries. But since then other fields of science than physics have developed ‘physics envy’ and they too are now busy searching for explanations. For example, chemistry finds its explanations in physics. And explanations in biology are found, on one level, in evolution theory and, on another level, in chemistry and physics.

I differ with Rutherford, though, in his narrow conception of science. To be sure, science involves searching for explanations. But production and collection of data through experimentation and observation and classification of the data supply the raw material for science to attempt to explain. Without them there would be nothing to explain and no ‘real science’ in Rutherford’s sense. So I include butterfly collecting in my broad conception of science.

The point of all that, for the purpose of this book, is to lead to the notion that science – even in its broad conception – not only makes much use of symmetry, but is essentially and fundamentally based on symmetry. Indeed, science rests firmly on the triple foundation of reproducibility, predictability, and reduction, all of which are symmetries, with additional support from analogy and objectivity, which are symmetries too. So it is not much of an exaggeration to claim that science is symmetry. Or perhaps in somewhat more detail, science is our view of nature through symmetry spectacles. That is one component of the main thesis of this book.

In addition to an exposition and justification of this central idea, that science is founded on symmetry, we also look into how symmetry is used in science in general and in physics in particular (Rutherford’s ‘real science’). And we find: symmetry of evolution (symmetry of the laws of nature), symmetry of states of physical systems, gauge symmetry of the fundamental interactions, and the symmetry inherent to quantum theory. So not only do we *view* nature through symmetry spectacles, but we *understand* nature in the language of symmetry. That is another component of this book’s main thesis.

All that leads to deep questions that await clarification. What is the source of all this symmetry? What is nature telling us? Is nature symmetry, at least in some sense? If not at the level that physics is presently investigating, are deeper levels of reality involved with symmetry in a very major way? Or even, will symmetry turn out to be what those fundamental levels are *all* about? Is symmetry the foundational principle of the Universe?

Such ideas lurk in the back of many physicists’ minds, and some physicists express them outrightly. Brian Greene, for one, states in Chap. 8 of [1]: “From our modern perspective, symmetries are the foundation from which laws spring.” And Stenger [2] adds his vote.

Speaking of the Universe, it is shown in this book that the Universe cannot possess exact symmetry. This connects to conceptual problems with symmetry breaking at ‘phase transitions’ in the evolution of the

Universe according to big-bang type cosmological schemes. Such and related matters are discussed, including the nature of the ‘quantum era’ that is assumed to form the first evolutionary stage in big-bang type schemes. But many questions remain for future elucidation. Are big-bang type cosmological schemes the best models for the evolution of the Universe? If so, did the Universe pass through distinct eras separated by transitions that might be characterized as ‘phase transitions’? What were the properties of the eras and of the transitions? Was there a ‘quantum era’? If there was, can it be meaningfully described? And can present-day high-energy physics reflect the properties of earlier stages in the evolution of the Universe? If it can, what will the results of experiments soon to be performed at high-energy laboratories, such as CERN’s Large Hadron Collider, reveal about the earlier Universe? And what will they tell us about today’s physics? Will they help clarify or will they sow confusion?

Here is the order of presentation: We start in Chap. 1 with a brief introduction to the concept of symmetry, including an analysis of the intimate relation between symmetry and asymmetry – especially that symmetry implies asymmetry – and a discussion of analogy and classification as symmetry. We then see in Chap. 2 what science is, how it makes use of symmetry, and how it is based solidly on symmetry. So solidly, in fact, that one might well view science as symmetry. In Chap. 3 we consider a number of ways in which physics, in particular, additionally makes use of symmetry. Since physics underlies the other sciences, we find that science is based even more solidly on symmetry, and perhaps nature will turn out to possess a symmetry foundation as well. The symmetry principle, also known as Curie’s principle, is derived in its various versions in Chap. 4. We see in Chap. 5 two ways in which the symmetry principle is very usefully applied in science. In Chaps. 6 and 7 we discuss the ideas of imperfect symmetry and symmetry in general and as applied to the Universe and its evolution, as well as related ideas.

There then follows the more formal part of the book, in which we develop a formalism of symmetry. Chapters 8 and 9 form a brief introduction to group theory, the mathematical language of symmetry, which is indispensable for serious quantitative, as well as qualitative, applications of symmetry in science, mostly in physics and chemistry. Nevertheless, in spite of that indispensability, Chaps. 8 and 9 can be skipped without too much harm to those preferring a more conceptual approach. Chapter 10 develops the language and formalism that underlie the application of symmetry. Group theory is unavoidable there,



but I try to allow the reader to make sense of the ideas even without group theory. And finally, in Chap. 11 we apply symmetry considerations and the symmetry formalism to physical processes and derive the symmetry principles that apply to them.

Chapter 12 brings together and summarizes the principles of symmetry that are developed and presented in this book.

I would like to express my thanks to my friends and colleagues Avshalom C. Elitzur and Lawrence W. Fagg, who kindly read the manuscript of this book and helped me with their comments and suggestions. And especially, I thank my wife, Mira Frost, for her unflagging support and for putting up with my disappearances into my study to work on this book.

Rockville, Maryland,  
August 2007

*J. Rosen*

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