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# Chiral Soliton Models for Baryons

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## Preface

The purpose of this monograph is to explain and review the chiral soliton picture for baryons and their low-energy properties. Even though this picture by now ages almost half a century, it is currently more than ever under intense investigation. Various revivals have let the model stay modern. Examples that initiated renewed interest are the quark spin contribution to the nucleon spin (“proton spin puzzle”) or the quest for pentaquarks and other exotic baryons.

Various motivations for soliton models can be thought of. Mostly they relate to the observed flavor and chiral symmetries of strong interactions and properties of quantum chromodynamics (QCD) when it is generalized to contain infinitely many color degrees of freedom. The author is fully aware that there is anything but an inevitable derivation of the soliton picture from QCD. This probably is a common characteristic of any model attempting to describe baryon properties at low energies. Chiral soliton models certainly do have their limitations. However, they definitely possess a degree of straightforwardness uncommon to other models for hadrons. It is the author’s hope that the reader will appreciate the attractive beauty resulting thereof. Quite a number of arguments and conclusions presented in this monograph reflect the author’s personal opinion. Yet, the interested reader should be able to gain an objective point of view from the comprehensive list of references that is included.

There are actually many variants of soliton models on the market: starting from the famous Skyrme model of pion fields via vector meson extensions to bosonized formulations of the quark flavor dynamics. They will all be discussed here. Though different variants highlight different issues, it should become clear that they have more features in common than in distinction. In particular, the comprehensive discussion on solitons in models for the quark flavor dynamics (Chaps. 2 and 3) is intended to demonstrate that quark and soliton models have indeed a common base. Even though actual explorations in the soliton picture differ considerably from those in quark models, to a large extent these differences just reflect the use of different field variables.

Some of the topics discussed here have already been reviewed in detail elsewhere. Nevertheless, it might be illuminating to get a different view on similar issues. In addition there are issues that have not been reviewed so far and they motivate this monograph all the more.

Not all the detailed and lengthy calculations will be made explicit. However, the tools provided should enable the interested reader to follow the original research articles or perform the computations independently. Some basic knowledge of quantum field theory, including its path integral formulation, is presupposed. It is also assumed that the reader has some basic knowledge of the representations of the groups  $SU(2)$  and  $SU(3)$ .

These lecture notes distinguish two styles. Chapters 1 through 6 discuss the basics of the soliton model for baryons, i.e., the motivation, the existence of solitons and their interpretation as baryons. These chapters are very detailed and with the help of the appendices the interested reader should be able to redo all the relevant calculations. In particular, beginners in the field will hopefully find this part of the monograph illuminating since one of its major purposes is to cover the gap between standard textbooks and current research. Chapter 1 introduces the subject. The following two chapters review the motivation of soliton models from the quark flavor dynamics. Here we will focus on the Nambu–Jona–Lasino model and explain how the soliton picture emerges from a microscopic quark model that contains all features of chiral symmetry. In Chap. 4 we will particularly examine the Skyrme model and also present the large- $N_C$  arguments that motivate this model. In Chaps. 5 and 6 we will discuss the quantization of the soliton to generate states with good baryon quantum numbers. In particular we will show in Chap. 6 that the baryon number one soliton must be quantized as a fermion. Effectively it is not possible to completely cover the voluminous amount of research that has been assembled in the field. Therefore the remaining chapters serve as survey on static baryon properties (Chap. 7), meson–baryon scattering (Chap. 8), exotic pentaquark baryons (Chap. 9) and systems with baryon number larger than one (Chap. 10). This review part should enable the reader to follow the original research papers that are vastly cited. This Monograph is round off with a short epilogue. A few appendices are included to facilitate comprehension of the calculations in the main body of this monograph.

Many people have contributed to the compilation of this monograph in various ways, e.g., direct collaborations and fruitful discussions over many years. This help is highly appreciated. I am afraid that the following list of names is incomplete: G. Holzwarth, J. Schechter, R. L. Jaffe, H. Reinhardt, H. Walliser, B. Schwesinger, A. Hayashi, N. W. Park, R. Alkofer, Ulf G. Meißner, L. Gamberg, N. N. Scoccola, E. Ruiz Arriola, M. Quandt, O. Schröder. Their insight and expertise has proven indispensable.

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