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Editors

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Rational Algebraic Curves

A Computer Algebra Approach

With 24 Figures and 2 Tables

Springer

RISC

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Preface

Algebraic curves and surfaces are an old topic of geometric and algebraic investigation. They have found applications for instance in ancient and modern architectural designs, in number theoretic problems, in models of biological shapes, in error-correcting codes, and in cryptographic algorithms. Recently they have gained additional practical importance as central objects in computer-aided geometric design. Modern airplanes, cars, and household appliances would be unthinkable without the computational manipulation of algebraic curves and surfaces. Algebraic curves and surfaces combine fascinating mathematical beauty with challenging computational complexity and wide spread practical applicability.

In this book we treat only algebraic curves, although many of the results and methods can be and in fact have been generalized to surfaces. Being the solution loci of algebraic, i.e., polynomial, equations in two variables, plane algebraic curves are well suited for being investigated with symbolic computer algebra methods. This is exactly the approach we take in our book. We apply algorithms from computer algebra to the analysis, and manipulation of algebraic curves. To a large extent this amounts to being able to represent these algebraic curves in different ways, such as implicitly by defining polynomials, parametrically by rational functions, or locally parametrically by power series expansions around a point. All these representations have their individual advantages; an implicit representation lets us decide easily whether a given point actually lies on a given curve, a parametric representation allows us to generate points of a given curve over the desired coordinate fields, and with the help of a power series expansion we can for instance overcome the numerical problems of tracing a curve through a singularity.

The central problem in this book is the determination of rational parametrizability of a curve, and, in case it exists, the computation of a good rational parametrization. This amounts to determining the genus of a curve, i.e., its complete singularity structure, computing regular points of the curve in small coordinate fields, and constructing linear systems of curves with prescribed intersection multiplicities. Various optimality criteria for rational parametrizations of algebraic curves are discussed. We also point to some applications of these techniques in computer aided geometric design. Many of the symbolic algorithmic methods described in our book are implemented in the program system CASA, which is based on the computer algebra system Maple.

Our book is mainly intended for graduate students specializing in constructive algebraic curve geometry. We hope that researchers wanting to get a quick overview of what can be done with algebraic curves in terms of symbolic algebraic computation will also find this book helpful.

This book is the result of several years of research of the authors in the topic, and in consequence some parts of it are based on previous research published in journal papers, surveys, and conference proceedings (see [ReS97a], [Sen02], [Sen04], [SeW91], [SeW97], [SeW99], [SeW01a], [SeW01b]).

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