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Local Newforms for $\mathrm{GSp}(4)$

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About the diagram. The diagram illustrates natural bases for the new- and oldforms in a generic representation π of $\mathrm{GSp}(4, F)$ with trivial central character. The solid dot in the first row is the newform at level N_π . The solid dots and circles of the k -th row represent vectors in a natural basis for the oldforms at level $N_\pi + k$. Thus, the dimension of the paramodular vectors at level N_π is 1, the dimension at level $N_\pi + 1$ is 2, the dimension at level $N_\pi + 2$ is 4, and so on. The basis at a particular level is obtained from the newform by application of the commuting level raising operators θ , θ' , and the self-dual operator η . The arrows pointing down and to the left correspond to θ , the arrows pointing down and to the right correspond to θ' , and the vertical arrows correspond to η . The black dots represent oldforms obtained solely by application of θ 's and θ' 's. The inner-most circles represent oldforms obtained by a single application of η , the circles immediately around the inner-most circles represent oldforms obtained by two applications of η , etc.

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