Theoretical and Mathematical Physics

The series founded in 1975 and formerly (until 2005) entitled *Texts and Monographs in Physics* (TMP) publishes high-level monographs in theoretical and mathematical physics. The change of title to *Theoretical and Mathematical Physics* (TMP) signals that the series is a suitable publication platform for both the mathematical and the theoretical physicist. The wider scope of the series is reflected by the composition of the editorial board, comprising both physicists and mathematicians.

The books, written in a didactic style and containing a certain amount of elementary background material, bridge the gap between advanced textbooks and research monographs. They can thus serve as basis for advanced studies, not only for lectures and seminars at graduate level, but also for scientists entering a field of research.

Howard J. Carmichael

Statistical Methods in Quantum Optics 2

Non-Classical Fields

With 89 Figures

Springer

Professor Howard J. Carmichael
University of Auckland
Physics Department
Private Bag 92019, Auckland, New Zealand
E-Mail: h.carmichael@auckland.ac.nz

Library of Congress Control Number: 200798040873

Originally published with the title "Texts and Monographs in Physics" with ISSN 0172-5998
ISSN 1864-5879
ISBN 978-3-540-71319-7 Springer Berlin Heidelberg New York

Springer is a part of Springer Science+Business Media
springer.com

Typesetting and production: LE-TEX Jelonek, Schmidt & Vöckler GbR, Leipzig, Germany
Cover: eStudio Calamar S.L., F. Steinen-Broo, Girona, Spain

SPIN 10720937 55/3180/YL - 5 4 3 2 1 0 Printed on acid-free paper

For Marybeth

Preface

I must admit with regret, and not a little embarrassment, that eight years have passed since I sat down to write a preface for the first volume of this book. A great deal has changed for the community of quantum opticians in the meantime. The interests of some have been turned to the fascinating properties of degenerate quantum gases where a number of analogies with quantum optics are to be found. Then there is the quantum information revolution: a whole new language to be learned, built around John Bell's reading of the Bohr–Einstein debate and venerable words like entanglement, launched in a new direction, with the goals of achieving an unbreakable code and a new paradigm for computation—a quantum-mechanical one. Considering the passage of time and what has occurred, I can only trust it will not disappoint to announce that this second volume of *Statistical Methods* has not been diverted in either direction—or, perhaps, rather closer to the truth, it could not: a path was already set in the preface to Volume 1, and this is the path I have followed in preparing Chaps. 9 through 19 of Volume 2.

The subtitle, *Nonclassical Fields*, is perhaps not as accurate as it might be as a summary of content; or to put it another way, if my aim from the start had been to write a book on this topic, parts of that book would differ significantly from what follows here. Possibly the most important thing missing, and something that should be said, is that there are two quite distinct paths to a definition of nonclassicality in quantum optics. The first is grounded in the existence, or otherwise, of a nonsingular and positive Glauber–Sudarshan P function. The physical grounding is in the treatment of optical measurements, specifically the photoelectric effect: for a given optical field, can the photoelectron counting statistics, including all correlations, be reproduced by a Poisson process of photoelectron generation driven by a classical light intensity, allowed most generally to be stochastic? Viewed at a more informal level, the question asks whether or not the infamous proposal of Bohr, Kramers, and Slater for the interaction of classical light and quantized atoms can be upheld in the presence of the observable photoelectron counting statistics.

This criterion for nonclassicality is likely to be the one offered up by most quantum opticians when pressed for a definition. There is, however, a second. It is an outgrowth of John Bell's work and does not speak directly about measurements of any sort. At issue are the variances and covariances of a set of quantum mechanical observables—the quadrature amplitudes occupying this or that optical mode: can these quantities all be computed from a classical probability distribution, admitting hidden variables but no nonlocal connections between the values they take? Generally speaking, but not always, variances and covariances computed from an entangled state within quantum mechanics cannot be recovered from a classical distribution. Squeezed light provides a notable counterexample; for it, a positive definite Wigner function serves as the required classical probability distribution. Thus, by the second Bell-based criterion, squeezed light is not nonclassical. (Though in a perverse reversal of Bell's argument, the entangled character of the two-mode squeezed state is often seen to trump this observation.) Squeezed light is of course nonclassical by the former P function criterion.

In this volume squeezed light is nonclassical. The "Nonclassical" of the subtitle is to be read in the P function sense. Starting with two chapters on squeezing in the degenerate parametric oscillator, the volume continues on with the theme taken up in Volume 1 of "methods developed in quantum optics for analyzing quantum fluctuations in terms of a visualizable evolution over time." These are the methods of the quantum–classical correspondence: the phase-space representations, which when applied to an operator master equation yield a Fokker–Planck equation, albeit, in many cases, only after a system size expansion of the full equation of motion is made—i.e., only when the quantum noise is sufficiently small. Applied to the degenerate parametric oscillator, the methods fail, though the positive P representation of Drummond and Gardiner does manage to resurrect "a visualizable evolution over time"—qualified, however, by serious difficulties of a new kind.

Chapters 9 and 10 deal with squeezing, the degenerate parametric oscillator, and how squeezed light generation causes the standard phase-space methods to fail. Chapter 11 then develops the positive P approach, while Chap. 12 uncovers the problems it encounters when the system size expansion no longer holds.

Problems with the positive P representation aside, much of the appeal of the phase-space approach is lost when the system size expansion fails. Its very premise is a classical dynamic plus quantum fluctuation "fuzz," the "fuzz" a perturbation by definition; "fluctuation" is defined in a classical sense from the very beginning. While the positive P representation escapes this background to some extent, it also retreats from all but a formal connection with the physics—as a generator of quantum averages—and any resolution of its difficulties can only deepen that retreat.

On this score it is worthwhile to recall my appeal in the preface to Volume 1: “Nothing in the Schrödinger equation fluctuates. What then *is* a quantum fluctuation?” A classically inspired method for computing quantum averages is unlikely to illuminate this question. The seven chapters from Chap. 13 to Chap. 19 work towards an outlook that possibly can.

The context for the development is provided by cavity QED, which is explored in Chaps. 13–16. Its defining conditions of strong dipole coupling between a resonant atomic transition and an optical cavity mode are essentially the same—for single atoms—as those defining a small system size, such that the system size expansion fails, and experiments have reached a remarkable level of sophistication, a level hardly imagined as researchers set out to realize strong dipole coupling some 20 years ago.

My attempt to illuminate the “What *is* a quantum fluctuation?” question occupies Chaps. 17–19. Here quantum trajectory theory is developed. The approach, at bottom, is conventional, recalling observations that have been made about the meaning of quantum mechanics since the time of Niels Bohr. Certainly nothing fluctuates in the Schrödinger equation; indeed, the Schrödinger equation describes no realized happenings of any sort—no realized events; it governs the time evolution of probabilities of events. To actually *realize* events, the probabilities must be put into action, to play out as a stochastic process. But here is the sticking point: the playing out is not unique, not only in the trivial sense that the throwing of a die yields different answers on every throw, but because the very shape of the die is not uniquely defined from within the Schrödinger equation itself. It is we the commentators who chose a shape through the question we chose to ask—or so it might appear, though in practice it is not so much a matter of commentators and their questions, but a subdivision of the physical world into a subsystem acting and one acted (irreversibly) upon. With only the “acting” subsystem defined, there are, of course, many possibilities for the subsystem “acted (irreversibly) upon” and such a division is not unique.

The many years that have passed since I began writing this book have left me indebted to numerous people, for their support and encouragement, and for the detection of many of those irritating errors that inevitably seem to make it into the typeset text. I thank both the University of Oregon and the University of Auckland for support during periods of concentrated work on the book. I am also indebted to the Alexander Humboldt Foundation, my German sponsor, Wolfgang Schleich, and his tireless wife Kathy, for their support during a year spent in Ulm; the visit allowed me to restart a project that had languished for quite some time. Then the patience of the editorial office of Prof. Wolf Beiglböck at Springer can only be wondered at. Finally, there are my students in Auckland, Mile Gu, Andy Chia, Changsuk Noh, Rob Fisher, and Felipe Dimer de Oliveira, who provided indispensable service by

reading parts of the text and detecting so many of those irritating errors, and my special thanks go to Hyunchul Nha who, as my postdoc, made numerous contributions that enabled me to improve what is written.

I must add that work on the book has stolen many hours away from my wife Marybeth. My principal debt is to her. We can both now be happy that this one cause, at least, of stolen hours is at a close.

Auckland Howard Carmichael
January 2007

Contents