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Flat Covers of Modules

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Dedicated to

Professor Edgar E. Enochs

My Mother Guizhen Zhu

and

My Father Youcheng Xu

Acknowledgement

Professor Enochs introduced me to the subject of flat covers five years ago. He also encouraged me to write this monograph two years ago when we made substantial progress in the theory of flat covers. I sincerely thank him for his constant encouragement and adivce. In fact, I have just reorganized and rewritten some of his work in many parts of this monograph.

I am grateful to the referees for their careful reading and useful suggestions. Using their reports and advice, I have been able to improve the original draft and remove some flaws.

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