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Multiple Integrals in the Calculus of Variations

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Multiple Integrals in the Calculus of Variations

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Preface

The principal theme of this book is "the existence and differentiability of the solutions of variational problems involving multiple integrals." We shall discuss the corresponding questions for single integrals only very briefly since these have been discussed adequately in every other book on the calculus of variations. Moreover, applications to engineering, physics, etc., are not discussed at all; however, we do discuss *mathematical* applications to such subjects as the theory of harmonic integrals and the so-called "$\bar{\partial}$-Neumann" problem (see Chapters 7 and 8). Since the plan of the book is described in Section 1.2 below we shall merely make a few observations here.

In order to study the questions mentioned above it is necessary to use some very elementary theorems about convex functions and operators on Banach and Hilbert spaces and some special function spaces, now known as "SOBOLEV spaces". However, most of the facts which we use concerning these spaces were known before the war when a different terminology was used (see CALKIN and MORREY [5]); but we have included some powerful new results due to CALDERON in our exposition in Chapter 3. The definitions of these spaces and some of the proofs have been made simpler by using the most elementary ideas of distribution theory; however, almost no other use has been made of that theory and no knowledge of that theory is required in order to read this book. Of course we have found it necessary to develop the theory of linear elliptic systems at some length in order to present our desired differentiability results. We found it particularly essential to consider "weak solutions" of such systems in which we were often forced to allow discontinuous coefficients; in this connection, we include an exposition of the DE GIORGI—NASH—MOSER results. And we include in Chapter 6 a proof of the analyticity of the solutions (on the interior and at the boundary) of the most general non-linear analytic elliptic system with general regular (as in AGMON, DOUGLIS, and NIRENBERG) boundary conditions. But we confine ourselves to functions which are analytic, of class C^∞, of class C^n_μ or C^n (see § 1.2), or in some Sobolev space H^m_p with m an integer ≥ 0 (except in Chapter 9). These latter spaces have been defined for all real m in a domain (or manifold) or on its boundary and have been used by many authors in their studies of linear systems. We have not included a study of these spaces since (i) this book is already sufficiently long, (ii) we took no part in this development, and (iii) these spaces are adequately discussed in other *books* (see A. FRIEDMAN [2], HORMANDER [1], LIONS [2]) as well as in many papers (see § 1.8 and papers by LIONS and MAGENES).

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Berkeley, August 1966

CHARLES B. MORREY, JR.

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