

Editors:

A. Dold, Heidelberg

F. Takens, Groningen

**Springer**

*Berlin*

*Heidelberg*

*New York*

*Barcelona*

*Budapest*

*Hong Kong*

*London*

*Milan*

*Paris*

*Santa Clara*

*Singapore*

*Tokyo*

Ke-Zheng Li Frans Oort

# Moduli of Supersingular Abelian Varieties



Springer

Authors

Ke-Zheng Li  
Graduate School of Academia Sinica  
Department of Mathematics  
P.O. Box 3908  
Beijing 100039, China  
e-mail: kzli@math07.math.ac.cn

Frans Oort  
Mathematisch Instituut  
Budapestlaan 6  
NL-3508 TA Utrecht, The Netherlands  
e-mail: oort@math.ruu.nl

Cataloging-in-Publication Data applied for

Die Deutsche Bibliothek - CIP-Einheitsaufnahme

**Li, Ke-Zheng:**  
**Moduli of supersingular Abelian varieties / Ke-Zheng Li ; Frans Oort. - Berlin ; Heidelberg ; New York ; Barcelona ; Budapest ; Hong Kong ; London ; Milan ; Paris ; Santa Clara ; Singapore ; Tokyo : Springer, 1998**  
(Lecture notes in mathematics ; Vol. 1680)  
ISBN 3-540-63923-3

Mathematics Subject Classification (1991):  
14K10, 14G15, 14L05, 14L15, 14D20, 14D22, 11G10, 11G15, 11R29

ISSN 0075-8434  
ISBN 3-540-63923-3 Springer-Verlag Berlin Heidelberg New York

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1998  
Printed in Germany

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typesetting: Camera-ready T<sub>E</sub>X output by the author  
SPIN: 10553445      46/3143-543210 - Printed on acid-free paper

## Contents

0. Introduction .....	1
1. Supersingular abelian varieties .....	11
2. Some prerequisites about group schemes .....	16
3. Flag type quotients .....	19
4. Main results on $\mathcal{S}_{g,1}$ (the principally polarized case) .....	24
5. Prerequisites about Dieudonné modules .....	28
6. PFTQs of Dieudonné modules over $W$ .....	35
7. Moduli of rigid PFTQs of Dieudonné modules .....	39
8. Some class numbers .....	51
9. Examples on $\mathcal{S}_{g,1}$ .....	55
10. Main results on $\mathcal{S}_{g,d}$ (the non-principally polarized case) .....	69
11. Proofs of the propositions on FTQs .....	73
12. Examples on $\mathcal{S}_{g,d}$ ( $d > 1$ ) .....	84
13. A scheme-theoretic definition of supersingularity .....	87
A. Appendix: Some historical remarks .....	96
References .....	106
Index .....	113