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The Blocking Technique, Weighted Mean Operators and Hardy's Inequality

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Für Klaus

Preface

The aim of these notes is to present a comprehensive treatment of the so-called blocking technique, together with applications to the study of sequence and function spaces, to the study of operators between such spaces, and to classical inequalities.

In these theories, and in other parts of Analysis, expressions of the form

$$\sum_{n=1}^{\infty}\left[a_n\left(\sum_{k=1}^{n}|x_k|^p\right)^{1/p}\right]^q$$

play an important rôle, most prominently perhaps in connection with Hardy's inequality. The analysis of such an expression, which we shall briefly call a *norm in section form*, has turned out to be demanding.

In many cases a problem becomes more accessible under a suitable renorming. Now, throughout the last four decades expressions of the form

$$\sum_{\nu=0}^{\infty}\left[\frac{1}{2^{\nu\alpha}}\left(\sum_{k\in I_\nu}|x_k|^p\right)^{1/p}\right]^q$$

have been appearing quite naturally in various parts of Analysis, very often in connection with coefficient conditions on series expansions of functions. Here, the I_ν form a partition of $\mathbb{N}$ into disjoint intervals, the most common partition being that into the dyadic blocks $[2^\nu, 2^{\nu+1})$. An expression of the above type is called a *norm in block form*.

It has already been noted by several authors that certain norms in section form can be replaced equivalently by a norm in block form. Such a renorming, which is referred to as the *blocking technique*, is of great practical value, for the analysis of norms in block form is much simpler: in many respects they behave just like the familiar l^p-norms.

In these notes we show that, apart from some trivial cases, in fact every norm in section form can be transformed into block form and, what is perhaps even more surprising, every norm in block form can be re-translated into section form. In that sense the blocking technique is universal. Chapter I provides a dictionary of transformations between the two kinds of norms. The related problem of characterising when two given norms are equivalent is of less relevance to the applications in these notes and is treated in the Appendix. In Chapter II we apply the blocking technique to study the structure of sequence spaces defined by norms in section form, while Chapter III contains applications to (generalised) weighted mean operators in l^p and to the weighted inequalities of Hardy and Copson.

It is more a matter of personal taste that we have chosen to concentrate our study on norms for sequences rather than on the corresponding integral norms for functions on the real line. In Chapter IV we indicate the integral analogues of our results.

Our research originated from a study of four papers by G. Bennett that revolve around the inequalities of Hardy and Copson. We have developed the blocking technique as a tool to attack some of his open problems. This has been successful; the solutions to three of his problems are contained in Sections 9, 10 and 17.

On the other hand, the results in Bennett's papers were instrumental in leading us to the appropriate transformations between section norms and block norms. Thus it is in two ways that these notes owe their existence to Grahame Bennett. I would therefore like to take this opportunity to express my sincere gratitude to him and my deep appreciation of the beauty of his work.

Hagen, October 1997 Karl-Goswin Grosse-Erdmann

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