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# Robust Synchronization of Chaotic Systems via Feedback



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To you, beauty that has shared your time  
and all my small conquests  
along recent years.  
Ricardo Femat

To Saïdy, Regina, Estela and Antonio.  
Gualberto Solís-Perales

# Preface

This pages include the results derived during last ten years about both suppression and synchronization of chaotic -continuous time- systems. Along this time, our concept was to study how the intrinsic properties of dynamical systems can be exploited to suppress and to synchronize the chaotic behavior and what synchronization phenomena can be found under feedback interconnection. Our findings have caused surprise to us and have stimulated our astonishing capability. Perhaps, reader can imagine our faces with opens eyes like children seeing around objects; which are possibly obvious for others and novel for us. A compilation of our surprises about these findings is being described along this book. Book contains both objectives to share our amazement and to show our perspective on synchronization of chaotic systems. Thus, while we were writing the preface, we discussed its scope. Thinking as a book readers, we found that a preface should answer, in few words, the following question: What can the reader find in this book?, reader can find our steps toward understanding of chaotic behavior and the possibility of suppressing and synchronizing it. We firstly show the chaos suppression form experimental domain to potential implementation in high tech system as a levitation system based on High Temperature Superconductors (HTS). This chapter is used as departing point towards a more complicated problem the chaotic synchronization. Then, reader travels by the synchronization of the chaotic behavior world throughout distinct feedback approaches. An extension to classical feedback is first ste, where a classification of synchronization phenomena and potential implementation allow to go beyond the control application. Then, a small trip by geometrical tools to induce synchrony on chaos behavior leads us back to engineering. Thus, we approach landing onto time-discretization of feedback towards potential implementation in microcontrollers. Finally, extension on geometrical tools and synchronization are included in last chapter.

April 2008

Ricardo Femat  
Gualberto Solís Perales

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