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Fuzzy Logic in Action: Applications in Epidemiology and Beyond

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Laécio Carvalho de Barros,  
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# Fuzzy Logic in Action: Applications in Epidemiology and Beyond



Springer

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*To those who contribute with their work  
to the control of the transmission of  
infectious diseases in the world*

*To Mara, Laila and Jamile*

*To my sweet love Eduardo Rissi and  
our little flowers Ana Clara and Mariana*

*To Cristina, Otávio and Luiza*

*To Dita, Moyses, Noel, Ivan  
Paula and Arthur*

# Foreword

Over four decades have passed since Lotfi Zadeh introduced the notion of fuzzy sets. During this period we have seen an impressive growth of conceptual, theoretical and methodological developments and a variety of applications in many fields. Current machinery in areas such as information storage and search, image processing and understanding, pattern recognition and control, computational biology and bioinformatics, to mention a few, have benefited considerably from the developments in fuzzy set theory. What is less visible is the paradigmatic change in the contemporary medical sciences, especially the changes that concern uncertainty, imprecision and vagueness. Despite the naturally imprecise character of the biological variables, only recently uncertainty, imprecision and vagueness have been viewed as essential, not only unavoidable, but in fact of a great practical utility. This is because uncertainty, imprecision and vagueness help to reduce complexity and to increase credibility and tractability, items that tend to maximize the usefulness of biological models.

Fuzzy Logic in Action: Applications in Epidemiology and Beyond, co-authored by Eduardo Massad, Neli Ortega, Laécio Barros, and Claudio Struchiner is a remarkable achievement. The book brings a major paradigm shift to medical sciences exploring the use of fuzzy sets in epidemiology and medical diagnosis arena. The volume addresses the most significant topics in the broad areas of epidemiology, mathematical modeling and uncertainty, embodying them within the framework of fuzzy set and dynamic systems theory. Written by leading contributors to the area of epidemiology, medical informatics and mathematics, the book combines a very lucid and authoritative exposition of the fundamentals of fuzzy sets with an insightful use of the fundamentals in the area of epidemiology and diagnosis. The content is clearly illustrated by numerous illustrative examples and several real world applications. Based on their profound knowledge of epidemiology and mathematical modeling, and on their keen understanding of the role played by uncertainty and fuzzy sets, the authors provide insights into the connections between biological phenomena and dynamic systems as a mean to predict, diagnose, and prescribe actions. An example is the use of

Bellman-Zadeh fuzzy decision making approach to develop a vaccination strategy to manage measles epidemics in São Paulo.

The book offers a comprehensive, systematic, fully updated and self-contained treatise of fuzzy sets in epidemiology and diagnosis. Its content covers material of vital interest to students, researchers and practitioners and is suitable both as a textbook and as a reference. The authors present new results of their own in most of the chapters. In doing so, they reflect the trend to view fuzzy sets, probability theory and statistics as an association of complementary and synergetic modeling methodologies.

Summing up, the authors have produced a book that presents a remarkably complete, well-organized, authoritative and reader-friendly exposition of the use of fuzzy sets in the area of epidemiology and diagnosis. The volume is a major contribution to a better understanding of how fuzzy set theory helps to manage complexity in biological phenomena and how to translate understanding into benefits for the human being.

Campinas, São Paulo, Brazil  
April 2008

Fernando Gomide  
University of Campinas

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