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Minimax and Monotonicity

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For Jacqueline

Preface

These notes had their genesis in three hours of lectures that were given in a "School" on minimax theorems that was held in Erice, Sicily in September — October, 1996. This was followed by an expanded version in five hours of lectures at the "Spring School" on Banach spaces in Paseky in the Czech Republic in April, 1997 which was followed, in turn, by an even more expanded version in ten hours of lectures at the University of Toulouse, France in May – June, 1997.

The lectures were initially conceived as three isolated applications of minimax theorems to the theory of monotone multifunctions. With each successive iteration, the emphasis gradually shifted to an examination of the "minimax technique", a method for proving the existence of continuous linear functionals on a Banach space, and to the relationship between this technique and monotone multifunctions. To this was finally added an attempt to collect together the results that have been proved on monotone and maximal monotone multifunctions on Banach spaces in recent years, and organize them into a coherent theory.

I would like to thank many people for their help and encouragement during the various stages of this project. I would first like to thank Biagio Ricceri for inviting me to Erice, Jaroslav Lukes, Jiri Kottas and Vaclav Zizler for inviting me to Paseky, and Jean–Baptiste Hiriart–Urruty for inviting me to Toulouse. I appreciate not only their excellent qualities as hosts, but also their grace and patience as audiences. Thanks are also due to Jonathan Borwein, Simon Fitzpatrick, Simeon Reich and Constantin Zalinescu for reading preliminary versions (or precursors) of these notes, and making many insightful comments and suggestions. I am especially grateful to Heinz Bauschke for reading a semi–final version of these notes from beginning to end, finding an amazing number of errors and ambiguities, and also for providing a number of excellent mathematical ideas. Last, but certainly not least, I would like to express my debt to Robert Phelps for his help and guidance all through this project. I appreciate his dogged insistence that I should try and make these notes as readable as possible. I would also like to acknowledge that his "Prague and Paseky" notes (which have been available electronically for several years) have been a source of inspiration.

Of course, despite all the excellent efforts of the people mentioned above, these notes doubtless still contain errors and ambiguities, and also doubtless have other stylistic shortcomings. At any rate, I hope that there are not too many of these. Those that do exist are entirely my fault.

Stephen Simons
May 28, 1998
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