Part I
Mathematical Preliminaries

The first part of the notes begins with an elementary and detailed exposition of the notion of a conformal transformation in the case of the flat spaces $\mathbb{R}^{p,q}$ (Chap. 1) and a thorough investigation of the conformal groups, that is the groups of all conformal transformations on the corresponding compactified spaces $N^{p,q}$ (Chap. 2). As a result, the conformal groups are finite-dimensional Lie groups except for the case of the Minkowski plane. In the case of the Minkowski plane one obtains (two copies of) the infinite dimensional Witt algebra as a complexified Lie algebra of infinitesimal conformal transformations.

Chapters 3 and 4 deal with central extensions of groups and Lie algebras. Central extensions occur in a natural way if one studies projective representations and wants to compare them with true representation in the linear space to which the projective space is associated. Since quantization represents observables as linear operators in a linear (mostly Hilbert) space W and the space of quantum states is the associated projectivation $\mathbb{P}(W)$, it is unavoidable that central extensions of Lie groups and Lie algebras naturally appear as the quantization of classical symmetries.

The first part of the notes concludes with an elementary description of the Virasoro algebra as the only nontrivial central extension of the Witt algebra (Chap. 5).

As a consequence, in a two-dimensional conformally invariant quantum field theory the Virasoro algebra shall be a symmetry algebra providing the theory with an infinite collection of invariants of motion.