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Probabilities on the Heisenberg Group

Limit Theorems and Brownian Motion

Springer

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Preface

Probability theory on algebraic and geometric structures such as e.g. topological groups has attracted much interest in the literature during the past decades and is a subject of growing importance. Stimuli which can not be overestimated for the research work which has and is currently been done in the field of probability theory on groups and related structures are the regular Oberwolfach conferences organized by L. Schmetterer, H. Heyer, and A. Mukherjea as well as the recent foundation of the "Journal of Theoretical Probability" also by A. Mukherjea.

In this work we will have, from the probabilistic point of view, a closer look at the so-called Heisenberg group. Its structure reflects the Heisenberg uncertainty principle as non-commutativity of the location and the momentum operator. In a certain sense, it is the simplest non-commutative Lie group, so it is clear that in generalizing classical results of probability theory to the non-commutative situation, one naturally passes by this group. Our aim will be to survey, under the limit theoretic aspect and its relation to Brownian motion, certain results which turned out to be valid on the Heisenberg group but which can not (or not yet) be generalized to the whole class of simply connected nilpotent Lie groups. For this wider framework, we refer (among others) to the forthcoming book of Hazod and Siebert (1995). So our work will to a certain degree be a complement to that book in the sense of some sort of a case study.

The second author of the above-mentioned book in preparation, Eberhard Siebert, untimely passed away in 1993. Without his fundamental contributions, the theory would at any rate not be at that level as it is now. It is one of the modest objectives of our book to underline the importance of Siebert's work in the development of probability theory on (in particular non-commutative) groups.

A word about applications: The Heisenberg group turned out to have many applications not only in mathematics itself (and there even in such remote fields such as combinatorics!), but also in physics (where it in fact comes from) and engineering science (signal theory). Due to the physical ignorance of the author, we have not tried to look for applications of the results presented in this work. The author would be delighted to hear one day about applications outside of "pure" mathematics!

It is my great pleasure to express my most sincere and heartfelt gratitude to my teacher and mentor Professor Henri Carnal for his constant benevolent support; to Professor Wilfried Hazod for his kind hospitality at the University of Dortmund; to Professor

René Schott for his kind hospitality at the University Henri Poincaré Nancy I; to Professor Yuri Stepanovič Hohlov and Professor Gyula Pap for many stimulating discussions; and last but not least to the Ingenieurschule Biel and its director Dr. Fredy Sidler for giving me the opportunity of taking a leave in order to continue my research activities and to begin with this work.

Biel-Bienne, May 1996

Daniel Neuenschwander

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