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Damir Filipović

Term-Structure Models

A Graduate Course

Springer

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for Susanne and Elena Christina

Preface

Changing interest rates constitute one of the major risk sources for banks, insurance companies, and other financial institutions. Modeling the term-structure movements of interest rates is a challenging task. One simple reason lies in the high dimensionality of this object, which is often assumed to be infinite. This creates a demand for mathematical models which differ from the standard stock market models. The origin of the term-structure models treated in this book can be traced back more than thirty years to the seminal work of Vasiček [160]. Since that time, the volume of traded interest rate sensitive derivatives has grown enormously.

This book gives an introduction to the mathematics of term-structure models in continuous time. It is suitable for a one-semester graduate course in mathematics, financial engineering, or quantitative finance. The focus is on a mathematically straightforward but rigorous development of the theory, which is illustrated with examples whenever possible. Each chapter ends with a set of exercises that provides a source for homework and exam questions. Readers are expected to be familiar with elementary Itô calculus, and analysis and probability theory on the level of e.g. Rudin [138] and Williams [161], respectively.

This book has emerged in several stages. I wrote the first version as lecture notes for a one-semester graduate course on fixed-income models in the fall term 2002/03 at the Department of Operations Research and Financial Engineering at Princeton University. The text has been gradually improved in subsequent lectures held at the Mathematics Institute at the University of Munich, at the Vienna Graduate School of Finance (VGSF), and at the Executive Academy of the Vienna University of Economics and Business Administration (WU). In the winter term of 2008/09 I completed the book by substantial revision and extension of the text, the inclusion of exercise and notes sections, and the addition of a completely new chapter on affine processes.

The number of books on term-structure models is rapidly growing, yet it is difficult, with a few exceptions, to find a convenient textbook for a one-semester graduate course on term-structure models for mathematicians and financial engineers. There are several reasons for this:

- Until recently, many textbooks on mathematical finance have treated stochastic interest rates as an appendix to the elementary arbitrage pricing theory, which usually requires constant (zero) interest rates.
- Interest rate theory is not as standardized as the arbitrage pricing models for stocks, such as the fundamental Black–Scholes model.
- The very nature of fixed-income instruments causes difficulties, other than for stock derivatives, in implementing and calibrating models. These issues should therefore not be left out.

Being aware that I must have overlooked important other contributions, I mention the following incomplete list of related books in alphabetic order: Björk [13] (introduction to mathematical finance, with a part on interest rate models), Brigo and Mercurio [27] (interest rate and credit risk models, practical implementation and calibration of selected models), Cairns [33] (a graduate course book on interest rate models), Carmona and Tehranchi [35] (mathematically advanced text on an infinite-dimensional analysis approach to interest rate models), James and Webber [100] (comprehensive resource on interest rate models, includes some historic account), Jarrow [103] (discrete-time introduction to interest rates), Musiela and Rutkowski [127] (comprehensive introduction to mathematical finance, with a large part on interest rate modeling and market pricing practice), Pelsser [131] (introduction to interest rate models and their efficient implementation), Rebonato [134] (emphasis on market practice for pricing and handling interest rate derivatives), Shreve [149] (introduction to mathematical finance with a chapter on term-structure models), and Zagst [163] (introduction to mathematical finance, interest rate modeling and risk management). In particular, more term-structure-related exercises, besides the set provided in this book, can be found in Björk [13], Cairns [33], and Shreve [149].

What distinguishes this book from others in particular is its comprehensive chapter on affine diffusion processes, which are among the most widely used factor models in finance. Another feature of this book is its section on the interplay between curve-fitting methods and factor models for the term-structure of interest rates.

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Vienna *Damir Filipović*

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