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Gianluca Fusai · Andrea Roncoroni

Implementing Models in Quantitative Finance: Methods and Cases

Springer

Gianluca Fusai

Dipartimento di Scienze Economiche
e Metodi Quantitativi
Facoltà di Economia
Università del Piemonte
Orientale "A. Avogadro"
Via Perrone, 18
28100 Novara
Italy
E-mail: gianluca.fusai@eco.unipmn.it

Andrea Roncoroni

Finance Department
ESSEC Graduate Business School
Avenue Bernard Hirsch BP 50105
Cergy Pontoise Cedex
France
E-mails: roncoroni@essec.fr
roncoroni@gmail.com

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To our families

To Nicola

Contents

Preface

Introduction

This book presents and develops major numerical methods currently used for solving problems arising in quantitative finance. Our presentation splits into two parts.

Part I is methodological, and offers a comprehensive toolkit on numerical methods and algorithms. This includes Monte Carlo simulation, numerical schemes for partial differential equations, stochastic optimization in discrete time, copula functions, transform-based methods and quadrature techniques.

Part II is practical, and features a number of self-contained cases. Each case introduces a concrete problem and offers a detailed, step-by-step solution. Computer code that implements the cases and the resulting output is also included.

The cases encompass a wide variety of quantitative issues arising in markets for equity, interest rates, credit risk, energy and exotic derivatives. The corresponding problems cover model simulation, derivative valuation, dynamic hedging, portfolio selection, risk management, statistical estimation and model calibration.

We provide algorithms implemented using either Matlab® or Visual Basic for Applications® (VBA). Several codes are made available through a link accessible from the Editor's web site.

Origin

Necessity is the mother of invention and, as such, the present work originates in class notes and problems developed for the courses "Numerical Methods in Finance" and "Exotic Derivatives" offered by the authors at Bocconi University within the Master in Quantitative Finance and Insurance program (from 2000–2001 to 2003–2004) and the Master of Quantitative Finance and Risk Management program (2004–2005 to present).

The "Numerical Methods in Finance" course schedule allots 14 hours to the presentation of Monte Carlo methods and dynamic programming and an additional 14 hours to partial differential equations and applications. These time constraints seem to be a rather common feature for most academic and professional programs in quantitative finance.

The "Exotic Derivatives" course schedule allots 14 hours to the introduction of pricing and hedging techniques using case-studies taken from energy and commodity finance.

Audience

Presentations are developed at an intermediate-advanced level. We wish to address those who have a relatively sound background in the theoretical aspects of finance, and who wish to implement models into viable working tools.

Users typically include:

A. Junior analysts joining quantitative positions in the financial or insurance industry;
B. Master of Science (MS) students;
C. Ph.D. candidates;
D. Professionals enrolled in programs for continuing education in finance.

Our experience has shown that, instead of more "novel-like" monographs, this audience usually succeeds with short, precise, self-contained presentations. People also ask for focused training lectures on practical issues in model implementation. In response, we have invested a considerable amount of time in writing a book that offers a "hands-on" educational approach.

Prerequisites

We assume the user is acquainted with basic derivative pricing theory (e.g., pay-off structuring, risk-neutral valuation, Black–Scholes model) and basic portfolio theory (e.g., mean-variance asset allocation), standard stochastic calculus (e.g., Itô formula and martingales) and introductory econometrics (e.g., linear regression).

Style

We strive to be as concise as possible throughout the text. This helps us minimize ambiguities in the methodological part, a pitfall that sometimes arises in nontechnical presentations of technical subjects. Moreover, it reflects the way we covered the presented material in our courses. An exception is made for chapters on copulas and Laplace transforms, which have been included due to their fast-growing relevance to the practice of quantitative finance.

We present cases following a constructive path. We first introduce a problem in an informal way, and then formalize it into a precise problem statement. Depending on the particular problem, we either set up a model or present a specific methodology in a self-contained manner. We proceed by detailing an implementation procedure, usually in the form of an algorithm, which is then coded into a programming language. Finally, we discuss empirical results stemming from the execution of the corresponding code.

Our presentation is modular. Thus, chapters in Part I offer systematic and self-contained presentations coupled with an extensive bibliography of published articles, monographs and working papers.

For ease of comparison, the notation adopted in each case has been kept as close as possible to the one employed in the original article(s). Note that this choice requires the reader to have a certain level of flexibility in handling notation across cases.

What's missing here?

By its very nature, a treatment on numerical methods in finance tends to be encyclopedic. In order to prevent textual overflow, we do not include certain topics. The most apparent missing topic is perhaps "discrete time financial econometrics". We insert a few cases on basic and advanced econometrics, but ultimately direct the reader to other more comprehensive treatments of these issues.

Content

Part I: Methods

Static Monte Carlo; Dynamic Monte Carlo; Dynamic Programming for Stochastic Optimization; Finite Difference Methods; Numerical Solution of Linear Systems; Quadrature Methods; The Laplace Transform; Structuring Dependence Using Copula Functions.

Part II: Cases

Portfolio Selection: 'Optimizing an Error'; Alpha, Beta and Beyond; Automatic Trading: Winning or Losing in a kBit; Estimating the Risk Neutral Density; An 'American' Monte Carlo; Fixing Volatile Volatility; An Average Problem; Quasi-Monte Carlo; Lookback Options: A Discrete Problem; Electrifying the Price of Power; A Sparkling Option; Swinging on a Tree; Floating-Rate Mortgages; Basket Default Swaps; Scenario Simulation using Principal Components; Parametric Estimation of Jump-Diffusions; Nonparametric Estimation of Jump-Diffusions; A Smiling GARCH.

The cases included are not necessarily a mechanical application of the methods developed in Part I. Conversely, some topics in Part I may not have a direct application in cases. We have, nevertheless, decided to include them both for the sake of completeness and given their importance in quantitative finance. We selected cases based on our research interests and (or) their importance in the practice of quantitative finance. More importantly, all methods lead to nontrivial implementation algorithms, reflecting our ambition to deliver an effective training toolkit.

Use

Given the modular structure of the book, readers can use its content in several ways. We offer a few sample sets of coursework for different types of users:

A. Six Hour MS Courses

A1. Quadrature methods for finance

Chapter "Quadrature Methods" (Newton–Cotes and Gaussian quadrature); inversion of the characteristic function and the Fast Fourier Transform (FFT); pricing using Lévy processes.

A2. Transform methods

Laplace and Fourier transforms; examples on pricing using Lévy processes and the CIR model; cases "Fixing Volatile Volatility" and "An Average Problem".

A3. Copula functions

Chapter "Structuring Dependence Using Copula Functions". Case "Basket Default Swaps".

A4. Portfolio theory

Cases "Portfolio Selection: Optimizing an Error", "Alpha, Beta and Beyond" and "Automatic Trading: Winning or Losing in a kBit".

A5. Applied financial econometrics

Cases "Scenario Simulation Using Principal Components", "Parametric Estimation of Jump-Diffusions", "Nonparametric Estimation of Jump-Diffusions" and "A Smiling GARCH".

B. Ten to Twelve Hour MS Courses

B.1. Monte Carlo methods

Chapters "Static Monte Carlo" and "Dynamic Monte Carlo". Cases "An 'American' Monte Carlo", "Lookback Options: A Discrete Problem", "Quasi-Monte Carlo", "A Sparkling Option" and "Basket Default Swaps".

B.2. Partial differential equations

Chapters "Finite Difference Methods" and "Numerical Solution of Linear Systems"; Cases "An Average Problem" and "Lookback Options: A Discrete Problem".

B.3. Advanced numerical methods for exotic derivatives

Chapters "Finite Difference Methods" and "Quadrature Methods"; Cases "An Average Problem", "Quasi-Monte Carlo: An Asian Bet", "Lookback Options: A Discrete Problem", and "A Sparkling Option".

B.4. Problem solving in quantitative finance

Presentation of various problems across different areas such as derivative pricing, portfolio selection, and financial econometrics; key cases are "Portfolio Selection: Optimizing an Error"; "Alpha, Beta and Beyond"; "Estimating the Risk Neutral Density"; "A Sparkling Option"; "Scenario Simulation Using Principal Components"; "Parametric Estimation of Jump-Diffusions"; "Nonparametric Estimation of Jump-Diffusions"; "A Smiling GARCH".

Abstracts

Portfolio Selection: Optimizing an Error

We assess the impact of sampling errors on mean-variance portfolios. Two alternative solutions (shrinkage and resampling) to the resulting issue are proposed. An out-of-sample comparison of the two methods is also presented.

Alpha, Beta and Beyond

We compare statistical procedures for estimating the beta coefficient in the market model. Statistical procedures (OLS regression, shrinkage, robust regression, exponential smoothing, Kalman filter) for measuring the Value at Risk of a portfolio are studied and compared.

Automatic Trading: Winning or Losing in a kBit

We present a technical analysis strategy based on the cross-over of moving averages. A statistical assessment of the strategy performance is developed using a nonparametric procedure (bootstrap method). Contrasting results are also presented.

Estimating the Risk-Neutral Density

We describe a lognormal-mixture based method to infer the risk-neutral probability density from option quotations in a given market. The model is tested by examining a trading strategy grounded on mispriced options.

An ‘American’ Monte Carlo

American option pricing requires the identification of an optimal exercise policy. This issue is usually cast as a backward stochastic optimization problem. Here we implement a forward method based on Monte Carlo simulation. This technique is particularly suited for pricing American-style options written on complex underlying processes.

Fixing Volatile Volatility

We propose a calibration of the celebrated Heston stochastic volatility model to a set of market prices of options. The method is based on the Fast Fourier algorithm. Extension to jump-diffusions and analysis of the parametric estimation stability are also presented.

An Average Problem

We describe, implement and compare several alternative algorithms for pricing Asian-style options, namely derivatives written on an average value in the Geometric Brownian framework.

Quasi-Monte Carlo: An Asian Bet

Quasi-Monte Carlo simulation is based on the fact that “wisely” selected deterministic sequences of numbers performs better in simulation studies than sequences produced by standard uniform generators. The method is presented and applied to the pricing of exotic derivatives.

Lookback Options: A Discrete Problem

We compare three algorithms (PDE, Monte Carlo and Transform Inversion) for pricing discretely monitored lookback options written on the minimum and the maximum attained by the underlying asset.

Electrifying the Price of Power

We illustrate a multi-agent competitive-equilibrium model for pricing forward contracts in deregulated electricity markets. Simulations are provided for sample price paths.

A Sparkling Option

A real option problem concerns the valuation of physical assets using a formal representation in terms of option pricing. We price co-generation power plants as an option written on the spark spread, namely the difference between electricity and gas prices.

Swinging on a Tree

A swing option allows the buyer to interrupt delivery of a given flow commodity, such as gas or electricity. Interruption can occur several times on a given time period. We cast this as a multiple-exercise American-style option and evaluate it using Dynamic Programming.

Floating Mortgages

An outstanding debt can be refinanced a fixed number of times over a larger set of dates. We compute the value of this option by solving for the corresponding multidimensional optimal stopping rule in a discrete time stochastic framework.

Basket Default Swaps

We price swaps written on a basket of liabilities whose default probability is modeled using copula functions. Alternative pricing methods are illustrated and compared.

Scenario Simulation Using Principal Components

We perform an approximate simulation of market scenarios defined by high-dimensional quantities using a reduction method based on the statistical notion of Principal Components.

Parametric Estimation of Jump-Diffusions

A simulation-based method for estimating parameters of continuous and discontinuous diffusion processes is proposed. This is particularly useful for asset valuation under high-dimensional underlying quantities.

Nonparametric Estimation of Jump-Diffusions

We estimate a jump-diffusion process using a kernel-based nonparametric method. Efficiency tests are performed for the purpose to assess the quality of the results.

A Smiling GARCH

We calibrate a GARCH model to the volatility surface by combining Monte Carlo simulation with a local optimization scheme.

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  Giovanna Boi (Consob, giovanna.boi@inwind.it)
  Andrea Bosio (Zero11 SRL, a.bosio@zero11.it)
  Paolo Carta (Royal Bank of Scotland plc, Paolo.CARTA@rbos.com)
  Gianna Figà-Talamanca (Università di Perugia, giannaft@unipg.it)
  Paolo Ghini (Green Energies, paolo.ghini@greenenergies.eu)
  Riccardo Grassi (MPS Alternative Investments SGR SpA, grassi@mpsalternative.it)
  Michele Lanza (Banca IMI, michele.lanza@bancaimi.it)
  Giacomo Le Pera (CREDARIS CPM, giacomo.lepera@credaris.com)
  Samuele Marafin (samuele.marafin@fastwebnet.it)
  Francesco Martinelli (Banca Lombarda, francesco.martinelli@bancalombarda.it)
  Davide Meneguzzo (Deutsche Bank, davide.meneguzzo@db.com)
  Enrico Michelotti (Dresdner Kleinwort, enrico.michelotti@dkib.com)
  Alessandro Moro (Morgan Stanley, alessandro.moro@morganstanley.com)
  Alessandra Palmieri (Moody's Italia SRL, alessandra.palmieri@moodys.com)
  Federico Roveda (Calyon, super fede <super_fede@email.it>)
  Piergiacomo Sabino (Dufenergy SA, piergiacomo.sabino@gmail.com)
  Marco Tarenghi (Banca Leonardo, marco.tarenghi@bancaleonardo.com)
  Igor Toder (Dexia, igor.toder@clf-dexia.com)
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Clearly, all errors, omissions and “bugs” are our own responsibility.

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