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Numerical Mathematics

Second Edition

With 135 Figures and 45 Tables

Springer

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Preface

Numerical mathematics is the branch of mathematics that proposes, develops, analyzes and applies methods from scientific computing to several fields including analysis, linear algebra, geometry, approximation theory, functional equations, optimization and differential equations. Other disciplines such as physics, the natural and biological sciences, engineering, and economics and the financial sciences frequently give rise to problems that need scientific computing for their solutions.

As such, numerical mathematics is the crossroad of several disciplines of great relevance in modern applied sciences, and can become a crucial tool for their qualitative and quantitative analysis. This role is also emphasized by the continual development of computers and algorithms, which make it possible nowadays, using scientific computing, to tackle problems of such a large size that real-life phenomena can be simulated providing accurate responses at affordable computational cost.

The corresponding spread of numerical software represents an enrichment for the scientific community. However, the user has to make the correct choice of the method (or the algorithm) which best suits the problem at hand. As a matter of fact, no black-box methods or algorithms exist that can effectively and accurately solve all kinds of problems.

One of the purposes of this book is to provide the mathematical foundations of numerical methods, to analyze their basic theoretical properties (stability, accuracy, computational complexity), and demonstrate their performances on examples and counterexamples which outline their pros and cons. This is done using the MATLAB® [1] software environment. This choice satisfies the two fundamental needs of user-friendliness and wide-spread diffusion, making it available on virtually every computer.

Every chapter is supplied with examples, exercises and applications of the discussed theory to the solution of real-life problems. The reader is thus in the ideal condition for acquiring the theoretical knowledge that is required to

[1] MATLAB is a trademark of The MathWorks, Inc.

make the right choice among the numerical methodologies and make use of the related computer programs.

This book is primarily addressed to undergraduate students, with particular focus on the degree courses in Engineering, Mathematics, Physics and Computer Science. The attention which is paid to the applications and the related development of software makes it valuable also for graduate students, researchers and users of scientific computing in the most widespread professional fields.

The content of the volume is organized into four Parts and 13 chapters.

Part I comprises two chapters in which we review basic linear algebra and introduce the general concepts of consistency, stability and convergence of a numerical method as well as the basic elements of computer arithmetic.

Part II is on numerical linear algebra, and is devoted to the solution of linear systems (Chapters 3 and 4) and eigenvalues and eigenvectors computation (Chapter 5).

We continue with Part III where we face several issues about functions and their approximation. Specifically, we are interested in the solution of nonlinear equations (Chapter 6), solution of nonlinear systems and optimization problems (Chapter 7), polynomial approximation (Chapter 8) and numerical integration (Chapter 9).

Part IV, which demands a mathematical background, is concerned with approximation, integration and transforms based on orthogonal polynomials (Chapter 10), solution of initial value problems (Chapter 11), boundary value problems (Chapter 12) and initial-boundary value problems for parabolic and hyperbolic equations (Chapter 13).
Part I provides the indispensable background. Each of the remaining Parts has a size and a content that make it well suited for a semester course.

A guideline index to the use of the numerous MATLAB programs developed in the book is reported at the end of the volume. These programs are also available at the web site address:

http://www1.mate.polimi.it/~calnum/programs.html.

For the reader's ease, any code is accompanied by a brief description of its input/output parameters.

We express our thanks to the staff at Springer-Verlag New York for their expert guidance and assistance with editorial aspects, as well as to Dr. Martin Peters from Springer-Verlag Heidelberg and Dr. Francesca Bonadei from Springer-Italia for their advice and friendly collaboration all along this project.

We gratefully thank Professors L. Gastaldi and A. Valli for their useful comments on Chapters 12 and 13.

We also wish to express our gratitude to our families for their forbearance and understanding, and dedicate this book to them.

Lausanne, Milan
January 2000

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Preface to the Second Edition

This second edition is characterized by a thourough overall revision.

Regarding the styling of the book, we have improved the readibility of pictures, tables and program headings.

Regarding the scientific contents, we have introduced several changes in the chapter on iterative methods for the solution of linear systems as well as in the chapter on polynomial approximation of functions and data.

Lausanne, Milan
September 2006

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Fausto Saleri

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