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Harald Grosse Ludwig Pittner (Eds.)

Low-Dimensional Models in Statistical Physics and Quantum Field Theory

Proceedings of the
34. Internationale Universitätswochen
für Kern- und Teilchenphysik
Schladming, Austria, March 4–11, 1995



Springer

Editors

Harald Grosse
Institut für Theoretische Physik, Universität Wien
Boltzmannngasse 5, A-1090 Wien, Austria

Ludwig Pittner
Institut für Theoretische Physik, Universität Graz
Universitätsplatz 5, A-8010 Graz, Austria

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These proceedings are dedicated to the memory of Professor Paul URBAN, founder and long-time director of the Schladming Winter School, who died at the age of ninety a few days before the start of this year's School.

Preface

This volume contains the written versions of the invited lectures and some of the seminars presented at the "34. Internationale Universitätswochen für Kern- und Teilchenphysik" in Schladming, Austria, which took place from March 4th to March 11th, 1995. The main contributions are centered around "Two-Dimensional Models of Quantum Field Theory and Statistical Physics". The lively developments within these subjects were also reflected by the many interrelated contributions.

The main items can be grouped into integrable (quantum) spin systems, which lead in the continuum limit to (conformal invariant) quantum field theory models and their algebraic structures, ranging from the Yang - Baxter equation and quantum groups to noncommutative geometry.

A review of integrable quantum spin models based on the reflection equation is given by P. Kulish from the St. Petersburg group. L. Faddeev uses this technology to quantize the Liouville model. V. Pasquier solves Calogero - Moser - type models of various forms. H. Grosse gives an introduction to this rapidly developing field. The Luttinger model as well as spin models such as the XX , XY , XXX and XXZ models are treated and their solution and interrelations sketched.

Nonrelativistic problems, from stability of matter to magnetic field problems and the Quantum Hall Effect (QHE), are treated by J. Fröhlich. In particular the fractional QHE is of great interest. Theoretical ideas behind it are connected to the Chern - Simons model, and edge currents are connected to conformal invariant models.

A. Alekseev tells us of a method to quantize the Chern - Simons model. Within the Hamiltonian approach, a two-dimensional system is obtained. The Poisson - Lie algebra of monodromies leads after deformation to a quantum group-invariant lattice model, whose representations are studied. Fusion rules lead to Verlinde's algebra.

The conformal symmetric two-dimensional models are reviewed by I. Todorov. Virasoro algebra representations were classified long ago, but the richness of these models is still not fully uncovered. Recently even number theoretic aspects have been used to study solutions of the Knizhnik - Zamolodchikov equation, describing the monodromies for n -point functions.

Since our low-dimensional models sometimes allow us to connect to physically realizable situations, we also invited E. Gornik to report on fascinating mesoscopic experiments. Modern technology and experiments with quantum wires and quantum dots are reported. Through tunneling, individual energy levels can be measured. The Luttinger model allows us to describe transport properties of quantum wires.

The algebraic aspects behind integrability led to the introduction of Lie - Poisson algebras, quadratic algebras, the classical Yang - Baxter relations, and deformations of these structures. N. Reshetikhin went through all these developments.

On any associative algebra, there exists (at least one) differential calculus. This interesting observation leads to many recent developments. It applies to quantum groups, but also to matrix algebras, as they occur within the Standard Model of elementary particles. The Higgs - effect is incorporated in a natural way. These developments are explained in detail by R. Coquereaux. The different approaches are compared.

Not only integrable models but also others are treated. K. Gawedzki tells us how to use the functional integration method to study problems of turbulence. A hierarchy of scales shows up. W. Thirring explains how the notion of Lyapunov exponents can be used for quantum systems too.

This year's Schladming School indicated lively development. A large number of physicists made this an impressive event.

In addition, excellent seminars were presented at the school, but due to space limitations not all of them could be included in these proceedings. However, a list of seminar contributions is given at the end of this volume, so that interested readers may request information or pertinent material directly from the authors.

Finally, we would like to express our thanks to the lecturers for all their efforts, to the sponsors of the school, above all the Austrian Ministry of Science and Research and the Government of Styria, for providing generous support, to our colleagues in the organizing committee for their assistance, and to Mrs. E. Neuhold for her help in bringing the files prepared by the authors in TEX or LATEX into their final form.

Graz, January 1996

H. Grosse and L. Pittner (Directors of the School)
V. Winkler (Scientific Secretary)

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List of Contributors

A.Yu. Alekseev

Institut für Theoretische Physik
ETH Zürich
Hönggerberg
CH-8093 Zürich
Switzerland
alekseev@itp.phys.ethz.ch

R. Coquereaux

Centre de Physique Theorique
CNRS - Luminy, Case 907
F-13288 Marseille, Cedex 9
France
coque@cpt.univ-mrs.fr

L.D. Faddeev

St.Petersburg Branch of
Steklov Mathematical Institute
Fontanka 27
St.Petersburg 191011
Russia

Research Institute for
Theoretical Physics
University of Helsinki
Siltavuorenpenger 20 C
SF-00014 Helsinki
Finland
faddeev@lomi.sph.su

K. Gawedzki

I.H.E.S., C.N.R.S.
F-91440 Bures-sur-Yvette
France
bongo@ihes.fr

H. Grosse

Institut für Theoretische Physik
Universität Wien
Boltzmannngasse 5
A-1090 Wien
Austria
grosse@pap.univie.ac.at

P.P. Kulish

St.Peterburgs Branch of
Steklov Mathematical Institute
Fontanka 27
St.Petersburg 191011
Russia

Depart. de Física Teór., IFIC
Centro Mixto Universidad
de Valencia - CSIC,
46100 Burjassot (Valencia)
Spain
kulish@evalvx.ific.uv.es

V. Pasquier

Service Physique Theorique
CEA-CEN Saclay
F-91191 Gif-sur-Yvette
France
pasquier@amoco.saclay.cea.fr

N. Reshetikhin

Department of Mathematics
University of California
Berkeley, CA 94720
USA
reshetik@math.berkeley.edu

W. Thirring

Institut für Theoretische Physik
 Universität Wien
 Boltzmannngasse 5
 A-1090 Wien
 Austria
 fwagner@pap.univie.ac.at

G. Amelino-Camelia

Center for Theoretical Physics
 MIT
 Cambridge, MA 02139
 USA
 amelino@mitlns.mit.edu

I. Andrić

Ruder Boskovic Institute
 POB 1016
 41001 Zagreb
 Croatia
 andric@thphys.irb.hr

A. Cappelli

INFN
 Largo E. Fermi 2
 I-50125 Firenze
 Italy
 cappelli@andrea.fi.infn.it

C. Gattringer

Max Planck-Inst. f. Physik
 Föhringer Ring 6
 D-80805 München
 Germany
 chg@dmumpiwh.bitnet

H. Johannesson

Inst. of Theoretical Physics
 Chalmers Univ. of Technology
 S-41296 Göteborg
 Sweden
 tfkhj@fy.chalmers.se

I.T. Todorov

Institute for Nuclear Research
 Bulgarian Academy of Sciences
 Tsarigradsko Chaussee 72
 BG-1784 Sofia
 Bulgaria
 toodorov@bgearn.bitnet

A. Koubek

Dept. of Appl. Math.
 and Theor. Physics
 University of Cambridge
 Silver Street
 CB3 9EW Cambridge
 United Kingdom
 a.koubek@damtp.cambridge.ac.uk

E. Langmann

Department of Theor. Physics
 Royal Institute of Technology
 S-10044 Stockholm
 Sweden
 langmann@theophys.kth.se

J. Lukierski

Institute of Theor. Physics
 PI. Maksa Borna 9
 University of Wroclaw
 50204 Wroclaw
 Poland
 lukier@proton.ift.uni.wroc.pl

G. Meissner

Inst. f. Theor. Physik
 Postfach 15 11 50
 Universität des Saarlandes
 D-66041 Saarbrücken
 Germany
 meissner@usb612.phys-th.uni-sb.de

P. Presnajder

Dept. of Theoretical Physics
Comenius University
Mlynská dolina
SK-84215 Bratislava
Slovakia
presnajder@fmph.uniba.sk

B.M. Zupnik

Bogoliubov Laboratory
of Theoretical Physics
JINR
141980 Dubna
Russia
zupnik@thsun1.jinr.dubna.su

A. Protogenov

Inst. of Appl. Physics
Russian Acad. of Sciences
46 Ulyanov Street
603600 Nizhny Novgorod
Russia
alprot@appl.nnov.su

P. Pyatov

Bogoliubov Lab. of
Theoretical Physics
JINR
141980 Dubna
Russia
pyatov@thsun1.jinr.dubna.su

J. Sobczyk

Institute of Theor. Physics
PI. Maksa Borna 9
University of Wroclaw
50204 Wroclaw
Poland

Th. Strobl

Inst. f. Theoret. Physik
RWTH-Aachen
Sommerfeldstr. 26-28
D-52056 Aachen
Germany
tstrobl@pluto.physik.rwth-aachen.de