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Jouni Suhonen

From Nucleons to Nucleus

Concepts of Microscopic Nuclear Theory

With 103 Figures

Springer

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To my dear family Tiina, Olli and Hannu

Preface

This book was born out of the need to gather and develop material for a two-term course on microscopic nuclear physics. As I started my Nuclear Physics II and III at the dawn of the present millenium, I realized that available material consisted either of qualitative introductory textbooks or of handbooks aimed for the professional practitioner. Neither of these two categories matched my idea of a guided pedagogical, hands-on approach to a quantitative description of the structure and decay of atomic nuclei.

The goal was to create a book that would contain an introduction to theory, worked-out examples and end-of-the-chapter exercises. At the same time the book should serve as a reference work for up-to-date applications of nuclear structure methods. With this vision in mind I set out to first produce hand-written lecture notes. On the next round of the two lecture courses the notes were transformed into typed hand-outs, which finally grew into this textbook.

This book builds on the premise that the reader has taken lecture courses on introductory nuclear physics and basic quantum mechanics. A good number of postgraduate and advanced undergraduate students, both theorists and experimentalists, have taken these courses. The style and contents of the book have been greatly influenced by their comments and criticism.

In each chapter I first derive the basic theoretical framework, apply it through worked-out examples and, in the end, discuss the physical implications and limitations of the theory. The formal derivations help understand the approximations and limitations behind the nuclear models that are introduced. However, the details of the derivations are not compulsory reading for someone who wants to go directly to the applications. In fact, in my lectures I skipped details of derivations and used the results as cookbook recipes, an approach particularly suitable for experimentally oriented students.

Even though the nuclear models introduced are generally valid, the examples and exercises are restricted to light and medium-heavy nuclei, up to the 0f-1p-$0g_{9/2}$ major shell. The reason for this is pedagogical: the small model space makes numerical problems tractable; a pocket calculator suffices in most cases. The applications are greatly simplified by numerical tables of two-body interaction matrix elements and of single-particle matrix elements of various electromagnetic and beta-decay operators. Even the most ambitious numerical applications amount to diagonalizing small symmetric matrices. In this way the calculational bulk does not obscure the physical insight to be gained from the exercises.

Only spherical nuclei are considered. However, nuclear structure, discussed in terms of particles, holes and quasiparticles, is boundlessly rich even under this constraint. Detailed single-particle and collective features emerge in electromagnetic and beta-decay transitions. The worked-out examples show in a tangible way how the quality of the computed nuclear wave functions can be probed by applying them to electromagnetic and beta-decay rates and comparing the results with experimental data.

As a textbook this work is self-contained. It introduces in the first four chapters the mathematical machinery needed for the theoretical derivations and their applications. The main theoretical tools are angular momentum coupling, tensor algebra, calculation of matrix elements of spherical tensor operators, the notion of the nuclear mean field and the subtleties of occupation number representation.

The book is divided into two parts. Part I is comprised of the first eleven chapters, and it deals with particles and holes. Chapters 5–7 discuss the simple single-particle shell model. At the same time details of electromagnetic and beta-decay transitions are introduced. Chapters 8–11 treat different configuration mixing schemes for particle, hole and particle–hole excitations in nuclei lying at the borders of closed major shells.

Chapters 12–19 form Part II of the book. They introduce Bogoliubov–Valatin quasiparticles and proceed to treat the quasiparticle mean field and configuration mixing of quasiparticle excitations. In all configuration mixing calculations I use the same two-body interaction, namely the surface delta interaction, SDI. The SDI is simple to derive and use, but at the same time it is realistic enough to do service in serious computing.

I wish to acknowledge the great help of my assistants Jussi Toivanen, Markus Kortelainen and Eero Holmlund, all PhDs by now, for their great pedagogical work in guiding the students through a good part of the exercises of this book. I owe my sincere thanks to Eero Holmlund also for producing the 100-odd figures of the book. I also want to extend my thanks to Dr. Matias Aunola, Mr. Heikki Heiskanen and Mr. Mika Mustonen for their contributions and corrections to the book.

A highly indispensable contribution to this book comes from Professor Emeritus Pertti Lipas, who initially guided my way to nuclear physics with his excellent lectures on the subject. As regards this book, he checked all the theory and worked-out numerical examples, corrected errors and introduced pedagogical improvements, revised the English language and style, and produced the final LaTeXtypescript including the demanding layout of equations and tables. Without doubt, Pertti Lipas is the person whom I owe the greatest debt for bringing this project to conclusion.

Finally, a project of this scale has demanded a great deal of extra time outside the office hours, time that I would normally have dedicated to my family. My deepest and loving thanks go to my wife Tiina and sons Olli and Hannu for their patience during the writing of my lecture notes and the final manuscript of this book.

Jyväskylä, Finland *Jouni Suhonen*
July 2006

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