

Andrzej Szepietowski

Turing Machines with Sublogarithmic Space

Springer-Verlag

Berlin Heidelberg New York

London Paris Tokyo

Hong Kong Barcelona

Budapest

Lecture Notes in Computer Science

843

Edited by G. Goos and J. Hartmanis

Advisory Board: W. Brauer D. Gries J. Stoer



Series Editors

Gerhard Goos
Universität Karlsruhe
Postfach 69 80
Vincenz-Priessnitz-Straße 1
D-76131 Karlsruhe, Germany

Juris Hartmanis
Cornell University
Department of Computer Science
4130 Upson Hall
Ithaca, NY 14853, USA

Author

Andrzej Szepietowski
Mathematical Institute, Gdansk University
Ul. Wita Stwosza 57, 80-952 Gdansk, Poland

Until March 1995:
FB 17 – Informatik, Universität GH Paderborn
Warburgerstraße 100, D-33095 Paderborn, Germany

CR Subject Classification (1991): F.1.1, F.4.1

ISBN 3-540-58355-6 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-58355-6 Springer-Verlag New York Berlin Heidelberg

CIP data applied for

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1994
Printed in Germany

Typesetting: Camera-ready by author
SPIN: 10475435 45/3140-543210 - Printed on acid-free paper

Preface

The main objects of investigation of space complexity theory are Turing machines with bounded space (the number of cells on the tape used during computation) and languages accepted by such machines. Although some important problems still remain open, much work has been done, and many exciting results have been obtained. Many of these results, however, have been proved under the assumption that the amount of available space is at least logarithmic with respect to the length of the input.

In this book I have presented the key result on space complexity, but concentrated on problems: What happens when we drop the assumption of at least logarithmic space and what do languages acceptable with sublogarithmic space look like?

The manuscript for this book was written at the Technical University of Gdansk and Gdansk University from 1991 to 1992. Parts of the manuscript were used as notes for lectures given at these universities. I want to thank the participants in these courses for their criticism. The final corrections to the manuscript were made during my stay at the University of Paderborn in 1994 under an Alexander von Humboldt Research Fellowship.

I would like to express my profound gratitude to Professor A.W. Mostowski for his encouragement and advice.

I owe a special debt to an anonymous referee for his or her helpful comments, to Mrs. K. Mostowska and J. Skurczyński for their careful reading of the original version of the manuscript, and to L. Chańko for helping me with typesetting the manuscript in TEX.

Paderborn, May 1994

Andrzej Szepietowski

Contents

1	Introduction	1
2	Basic Notions	7
2.1	Turing Machines	7
2.2	Configuration and Computation	9
2.3	Internal Configuration	11
2.4	Alternating Turing Machines	12
2.5	Space Complexity	13
3	Languages Acceptable with Logarithmic Space	15
3.1	Two Examples of Languages Acceptable with Logarithmic Space	15
3.2	Pebble Automata	16
3.3	$NSPACE(\log n)$ Complete Language	18
4	Examples of Languages Acceptable with Sublogarithmic Space	21
4.1	Languages Acceptable with $o(\log n)$ Space	21
4.2	Tally Languages Acceptable with Sublogarithmic Space	24
5	Lower Bounds for Accepting Non-regular Languages	27
5.1	Lower Bounds for Two-way Turing Machines	28
5.2	Lower Bounds for One-way Turing Machines	33
5.3	Lower Bounds for the Middle Mode of Space Complexity	34
6	Space Constructible Functions	37
6.1	Definitions and Basic Properties	37
6.2	Fully Space Constructible Functions	39
6.3	Nondeterministically Fully Space Constructible Functions	42
7	Halting Property and Closure under Complement	47
7.1	Halting Property of Turing Machines with Logarithmic or Greater Space	48
7.2	Closure under Complement of Strong Deterministic Complexity Classes	49
7.3	Closure under Complement of Strong Nondeterministic Space Complexity Classes above Logarithm	52
7.4	Closure under Complement for Bounded Languages Acceptable by Nondeterministic Turing Machines	53

VIII Contents

7.5	Nonclosure under Complement of Language Acceptable by Weakly Space Bounded Turing Machines.	55
8	Strong versus Weak Mode of Space Complexity	61
8.1	Weak and Strong Mode of Space Complexity for Fully Space Constructible Functions	61
8.2	Weak and Strong Complexity Classes above Logarithm	62
8.3	Weak and Strong Complexity Classes below Logarithm	63
8.4	Strong Mode of Space Complexity for Recognizing Machines	66
8.5	Weak and Middle Modes of Space Complexity above Logarithm	66
9	Padding	67
9.1	Padding above Logarithm	67
9.2	Padding below Logarithm	70
10	Deterministic versus Nondeterministic Turing Machines	77
10.1	Determinism versus Nondeterminism above Logarithm	77
10.2	Determinism versus Nondeterminism below Logarithm	78
11	Space Hierarchy	81
11.1	Diagonalization	81
11.2	Space Hierarchy below Logarithm	82
12	Closure under Concatenation	85
13	Alternating Hierarchy	89
13.1	Collapsing of the Alternating Hierarchy above Logarithm	90
13.2	Noncollapsing of the Alternating Hierarchy below Logarithm	92
14	Independent Complement	95
15	Other Models of Turing Machines	99
15.1	Two-dimensional Turing Machines	100
15.2	Inkdot Turing Machines	104
15.3	1-pebble Turing Machines	107
15.4	Demon Turing Machines	109
	References	111
	Subject Index	113
	Symbol Index	115