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PREFACE

During the week from the 24th to 29th of June 1985 the 14th International Conference on Differential Geometric Methods in Mathematical Physics was held under the auspices of the University of Salamanca. As is traditional in this series of Conferences, it was devoted "to an exchange between physics and mathematics in which differential geometric viewpoints play a predominant role".

The main topic of the Conference was "Mathematical problems in Classical Field Theory" with special emphasis on Superfield Theory, although communications concerning "Geometric Quantization" and "Symplectic techniques applied to Physics" were also offered since they are two topics strongly influenced by earlier DGM-Conferences.

We shall briefly summarize the main results presented in the 1985 Conference starting with the contributions devoted to Superfield Theory and related problems: super-algebras and supermanifolds, which was a top priority area, as mentioned, in this Conference.

The study of Lie superalgebras was one of the reasons for introducing the notion of supermanifoldsand this is currently an area that is arousing considerable interest. Sánchez-Valenzuela and Sternberg characterized the conformal superalgebra of ordinary space-time within the Hermitian superalgebras (introduced by Sternberg and Wolf) and gave a geometric interpretation of the group of automorphisms of the conformal super-algebra using certain Clifford algebras. By means of a weak triality principle, Crumeyrolle constructed Lie superalgebras associated with the geometry of Minkowski space, and provided the specific tables of the graded Lie brackets of its generators, thus finding models of different Lie superalgebras of physical interest. Batchelor established a functor from the category of closed immersions of ordinary differential manifolds to the category of graded manifolds and demonstrated that all graded manifolds are in the image of such a functor. In this way she reinterpreted graded Lie groups as pairs of ordinary Lie groups and described their superalgebras in terms of Lie algebras.

From amore physical point of view, Choquet-Bruhat offered the general form of a Lagrangian valid for a graded Einstein-Cartan theory in supermanifolds, with the field equations satisfied by its critical points and the identities that satisfy such equations due to the invariance of the Lagrangian by diffeomorphisms of the base manifold and Lorentz transformations of the tangent spaces. Bruzzo developed a variational calculus on fibered supermanifolds in the sense of DeWitt-Rogers and generalized the Utiyama and Noether theorems.

López Almorox generalized for the Kostant graded manifolds the bundle of graded connections and proved a Utiyama type theorem. Finally, Hernández Ruipérez and Muñoz Masqué established the notion of first-order variational problems for a regular projection of graded manifolds in terms of Berezin densities and showed that such problems are equivalent to higher-order variational problems defined by graded forms.

Other contributions to classical and quantized field theory were: Kerner dealt with Kaluza-Klein models which include the spontaneus symmetry breaking that comes from the Higgs fields, using combinations of the Riemann tensor, of the Ricci tensor and of the scalar curvature that lead to second-order field equations. In this way the Kaluza-Klein models allow certain generalizations of classical relativity.

Ne'eman offered a very complete development of the second quantized superstring, and gave a historial survey since its birth as a hadron bootstrap theory until current applications in quantum gravitational dynamics and super-unification.

Within this third block of contributions we should finally like to highlight those of Duval to the classical and quantized description of particles (galilean and and minkowskian) with $\frac{1}{2}$ spin and that of McCrea to the study and integration of the equations of the Poincaré gauge theory of gravitation by the application of computer algebra. Duval posed the problem of discovering how the Levy-Leblond equation arises in the Kostant-Souriau theory and how it is possible to explain, from first principles, its similarity with the Dirac equation.

To end, the contributions with a markedly more mathematical character are grouped in a fourth block:

Modugno introduced the notion of a system over double fibred manifolds, included examples to illustrate such a concept and paid special attention to systems of connections. He moreover proved that strong differential calculus permits a direct formulation of free fields satisfying Maxwell equations.

Pereira da Silva studied the representation, defined by the Lie derivative, of the Lie algebra $Der(M)$ of vector fields over a manifold M on the space Ω_2 of 2-forms over M and associated to this representation an essential extension of $Der(M)$ by Ω_2. Applications to unimodular manifolds, manifolds with a generalized foliation and Poisson manifolds were given.

J. Gomez and F. Varela demonstrated that if a compact, connected and orientable manifold has a volume form that is expressed globally in a certain way, owing to this fact the manifold's differentiable structure is defined and is diffeomorphic to a sphere or product of spheres.

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