

# Lecture Notes in Mathematics

Edited by J.-M. Morel, F. Takens and B. Teissier

## Editorial Policy

for the publication of monographs

1. Lecture Notes aim to report new developments in all areas of mathematics – quickly, informally and at a high level. Monograph manuscripts should be reasonably self-contained and rounded off. Thus they may, and often will, present not only results of the author but also related work by other people. They may be based on specialized lecture courses. Furthermore, the manuscripts should provide sufficient motivation, examples and applications. This clearly distinguishes Lecture Notes from journal articles or technical reports which normally are very concise. Articles intended for a journal but too long to be accepted by most journals, usually do not have this “lecture notes” character. For similar reasons it is unusual for doctoral theses to be accepted for the Lecture Notes series.

2. Manuscripts should be submitted (preferably in duplicate) either to one of the series editors or to Springer-Verlag, Heidelberg. In general, manuscripts will be sent out to 2 external referees for evaluation. If a decision cannot yet be reached on the basis of the first 2 reports, further referees may be contacted: the author will be informed of this. A final decision to publish can be made only on the basis of the complete manuscript, however a refereeing process leading to a preliminary decision can be based on a pre-final or incomplete manuscript. The strict minimum amount of material that will be considered should include a detailed outline describing the planned contents of each chapter, a bibliography and several sample chapters.

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Authors should also be aware that parallel submission of their manuscript to another publisher while under consideration for LNM will in general lead to immediate rejection.

3. Manuscripts should in general be submitted in English.

Final manuscripts should contain at least 100 pages of mathematical text and should include

- a table of contents;
- an informative introduction, with adequate motivation and perhaps some historical remarks: it should be accessible to a reader not intimately familiar with the topic treated;
- a subject index: as a rule this is genuinely helpful for the reader.

Continued on inside back-cover

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# Harmonic Functions on Groups and Fourier Algebras



Springer

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# Preface

The purpose of this monograph is to introduce some new aspects to the theory of harmonic functions and related topics. They are a fusion of some recent developments in non-associative functional analysis, semigroups and harmonic analysis. More specifically, we study the algebraic analytic structures of the space of bounded *complex* harmonic functions on a locally compact group  $G$  and its non-commutative analogue, the space of harmonic functionals on the Fourier algebra  $A(G)$ . We show that they are both the ranges of contractive projections on von Neumann algebras and therefore admit Jordan algebraic structures which are usually non-associative. This provides a natural setting to apply new methods and results from non-associative analysis, semigroups and Fourier algebras. We use these devices to study, among others, the Poisson representation of bounded complex harmonic functions on  $G$ , the semigroup structures of the Poisson space and the non-associative geometric structures of the harmonic functionals.

This work was done during several mutual visits of the authors at the University of Alberta and University of London, supported by EPSRC and NSERC grants. All results are new, some of which have been presented at seminars and workshops in London, Oxford, Edmonton, Hong Kong, Irvine, Toulouse and Oberwolfach. We thank warmly the audience at these institutions for their inspiration and hospitality. Above all, we are grateful to our families for their constant support and encouragement.

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