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Nash Manifolds

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PREFACE

The purpose of these Lecture Notes is to construct a theory of real manifolds equipped with "algebraic" structures. Real nonsingular algebraic varieties display apparently singular phenomena, and this suggests we should not expect a unified theory to emerge from them. As we shall see, however, the objects we consider in these Notes form a broader class than the real nonsingular algebraic varieties.

A function on $\mathbb{R}^n$ is algebraic if and only if it is analytic and if the graph is semialgebraic in $\mathbb{R}^n \times \mathbb{R}$. We call such a function a C^ω *Nash function*. We also call a C^r map with semialgebraic graph, $0 \leq r \leq \omega$, a C^r *Nash map*. Then we define a C^r *Nash manifold* by sticking finite semialgebraic open sets in $\mathbb{R}^n$ together along semialgebraic open subsets by C^r Nash diffeomorphisms. It is the category of C^r Nash manifolds and C^r Nash maps that we are interested in.

The compact case was already studied well. In [N] Nash showed that a compact C^1 manifold M can be imbedded in a Euclidean space $\mathbb{R}^n$ so that the image is a C^ω Nash submanifold of $\mathbb{R}^n$. He proved also that such a C^ω Nash manifold structure on M is unique up to C^ω Nash diffeomorphism. Hence we can endow a compact C^1 manifold with "algebraic" properties, which appears to contribute to differential topology. Indeed there are two applications [A-M] and [K]. Hoping for more applications, Palais developed a theory of compact affine C^ω Nash manifolds in [Pa].

We extend the object from C^ω Nash manifolds to C^r Nash manifolds. This is because there always exists a partition of unity of C^r Nash class if $0 \leq r < \infty$, a theorem of approximation of a C^r Nash map by a C^ω Nash map holds true, and because the similarity and the contrast between the C^0 Nash category and the C^ω Nash category are clear and interesting. Our theory is based mainly on results in $[Sh_1], \ldots, [Sh_5]$ and [S-Y].

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