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# Satellite Altimetry in Geodesy and Oceanography

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## FOREWORD

The International Summer School of Theoretical Geodesy is an Italian tradition, initiated by Antonio Marussi and carried on by me in recent years, with the very fundamental help of Reiner Rummel.

So it has been more than 4 years ago, with the course on "Theory of Satellite Geodesy and Gravity Field Determination", and even more this year with the course on "Satellite Altimetry in Geodesy and Oceanography".

The success of the school is absolutely due to the high scientific level and the bright personality of the teachers, who have nicely complemented and integrated each other, and the merit of putting them together is entirely of Reiner Rummel, who has been the true scientific organizer of the school.

The subject has been chosen years ago with the perspective of giving support to a research which was likely to grow, due to the foreseen (now flying) new satellite missions; the interdisciplinarity of the subject was a challenge that has been turned into a success by the spectacular competence of the teachers.

Also the practical organization of the school has worked very well, thanks first of all to the hospitality of the International Centre of Theoretical Physics, of its Deputy Director, Prof. Luciano Bertocchi, and its staff, in particular Miss Daniela Giombi; but the real soul of the organization has been Miss Elena Raguzzoni to whom go the sincere thanks of all the attendees as well as my personal congratulations.

Special thanks are also due to those organizations that have given us a real possibility to organize this course by supporting it financially, primarily creating fellowships for many students: so we want to acknowledge the support of IUGG and IAG (also thanks to their secretaries general, Dr. George Balmino and Dr. Claude Boucher) as well as the Italian firm Alenia.

Special thanks are also due to the Department of Geodesy of the University of Delft and to the Department of Environmental Engineering of the Politecnico of Milano, which have supplied a decisive support, substituting at the last moment other organizations, that traditionally helped the courses of this School, but this time did not.

Fernando Sansò

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