

# Lecture Notes in Control and Information Sciences

Edited by M.Thoma and A.Wyner

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V.L. Mehmann (Ed.)

## The Autonomous Linear Quadratic Control Problem

Theory and Numerical Solution

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Springer-Verlag  
Berlin Heidelberg New York  
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ISBN 3-540-54170-5 Springer-Verlag Berlin Heidelberg New York

ISBN 0-387-54170-5 Springer-Verlag New York Berlin Heidelberg

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Printed in Germany

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Printing: Mercedes-Druck, Berlin

Binding: B. Helm, Berlin

61/3020-543210 Printed on acid-free paper.

**For Lennart**

**Preface:** In this book a survey is given on the state of the art in theory and the numerical solution of general linear quadratic optimal control problems with differential algebraic equation constraints. This field is very active in the last ten years and in particular the step from standard ordinary differential equation constraints to differential algebraic equation constraints has received a lot of interest. Although the theory for the standard case is known since the sixties the development of numerical methods is not complete. For the descriptor case this is even more the case. Here not even the development of the theory can be considered near to complete. On the other hand these problems occur in many applications, in particular in engineering and it is therefore important to have a solid mathematical theory as well as reliable numerical algorithms. For this reason we review and extend theory and numerical methods for these problems together and we hope that this book helps increase the cooperation between pure mathematicians, numerical analysts and practioners.

The basis of this book was laid in my Habilitation thesis at the University of Bielefeld in 1987/1988. Apart from the survey of the current literature, it contains many new results and methods that were obtained with co-authors in the last 6 years. Without the joint efforts of these people this work would have never been finished. I wish to express my thanks to these co-authors, Gregory Ammar, Angelika Bunse-Gerstner, Ralph Byers, Ton Geerts, Gerd Krause, Peter Kunkel, Nancy Nichols and David Watkins and also to my students Engkan Tan, Ulrike Flaschka and Dagmar Zywietz and last but not least to Ludwig Elsner, for many helpful discussions.

I further wish to thank Regine Hollmann and Marion Matz for help in typesetting this book.

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