Lecture Notes in Mathematics 1473

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Functional Differential Equations with Infinite Delay

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Dedicated to Professor Taro Yoshizawa on his 70th birthday

PREFACE

The aim of this text is to deal with functional differential equations with infinite delay on an abstract phase space characterized by several axioms which are satisfied by different kinds of function spaces. The standard spaces which we have in mind are the one of continuous functions on $(-\infty,0]$ that are endowed with some restriction on their asymptotic behavior at $-\infty$, and the one of measurable functions on $(-\infty,0]$ that are integrable with respect to some Borel measure equipped with mild conditions. These spaces are carefully adopted as phase spaces for equations with infinite delay so as to solve each problem which we encounter in applications, nevertheless many fundamental properties of these equations hold good independently of the choice of phase spaces. The axiomatic approach is not only advantageous to summarize these properties, but also supply fruitful ideas and methods to investigate the mathematical structure which reflects the effect of infinite delay. Hence, we now intend to develop a unified theory of this field in terms of functional analysis and dynamical systems. For the sake of clear understanding of our ground, many elementary facts and proofs are given as completely as possible. But, few attempts had been made to give examples of equations in applications, and only a limited number of references are presented at the end of this text.

This text consists of nine chapters. Chapter 1 contains the formulation of axioms of the phase space together with many examples. Chapter 2 is a presentation of basic theory of existence, uniqueness, continuous dependence, etc. of solutions. After a brief introduction to Stieltjes integrals in Chapter 3, the theory of linear equations is developed from Chapter 4 through Chapter 6. Chapter 7 is an introduction to fading memory spaces. In Chapter 8, the stability problem in functional differential equations on a fading memory space is studied in connection with limiting equations. In succession, the existence of periodic and almost periodic solutions of functional differential equations is discussed in Chapter 9.

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