

# Lecture Notes in Mathematics

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## Multigrid Methods II

Proceedings of the 2nd European Conference  
on Multigrid Methods  
held at Cologne, October 1–4, 1985

Edited by W. Hackbusch and U. Trottenberg

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## Preface:

These proceedings contain a selection of papers presented at the

### 2nd European Conference on Multigrid Methods

which was held at the University of Cologne on October 1–4, 1985. It was organized by:

- Gesellschaft für Mathematik und Datenverarbeitung (GMD),  
Institut für Methodische Grundlagen,  
St Augustin–Birlinghoven,
- GAMM–Fachausschuß “Effiziente numerische Verfahren für  
partielle Differentialgleichungen”,

in cooperation with

- Mathematisches Institut der Universität zu Köln.

Whereas the First European Conference on Multigrid Methods in 1981 could be regarded as a first breakthrough of the multigrid idea to the field of application problems, this second conference reflected the systematic progress in the last four years in both theory and practice as well as the strongly grown acceptance of multigrid methods among users.

The papers of this volume are assumed to be of interest for a large number of scientists in academic and industrial research. They give a survey on the current multigrid development. (More specialized papers presented at the conference have been published in the GMD–Study no. 110; they are listed at the end of this volume.)

Most of the papers in this volume deal with problems arising from fluid– and aerodynamics. Special multigrid solvers are discussed for potential and Euler flows as well as for the solution of the full Navier–Stokes equations in highly complicated geometries. These problems, which are perhaps the most challenging ones for numerical mathematics of all, were also at the centre of interest at the conference.

New fields of applications considered in these proceedings are: statistical physics and magnetohydrodynamic problems. Furthermore these proceedings present papers, in which various “classical” numerical problems are treated, which are interesting in theoretical regard but which are also directly important for practical applications: local mesh refinements, error estimates, continuation techniques for nonlinear steady state problems, multigrid and conjugate gradients, indefinite problems and the treatment of singularities.

The growing importance of parallel computer architectures and – closely related to these – the need for highly efficient algorithms specially designed for this new type of computers have also introduced new aspects into multigrid development. Two of the papers in this book treat questions from this new field of multigrid research. They consider the performance of multigrid methods on special types of MIMD–architectures (bus–coupled systems and hypercubes).

We would like to express our gratitude to the Executive Board of the GMD and to the University of Cologne for all their immaterial and material support of the conference.

The practical organization was excellently carried out by Johannes Linden, Institut für Methodische Grundlagen of the GMD. He was supported by Wolfgang Joppich, several other coworkers, the secretaries and by the students of this institute. Furthermore, the Abteilung für Informationswesen of the GMD and the Institute of Mathematics of the University of Cologne provided substantial assistance to the organization of the conference. We would like to thank all persons involved.

Finally we thank all of the more than 130 participants from 12 countries and especially the lecturers for their contributions to the success of the conference.

St. Augustin, March 1986

Wolfgang Hackbusch  
Ulrich Trottenberg

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