

# Lecture Notes in Physics

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H. Araki

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H. Gausterer C. B. Lang (Eds.)

# Computational Methods in Field Theory

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## **Editors**

H. Gausterer

C. B. Lang

Institut für Theoretische Physik

Karl-Franzens-Universität

Universitätsplatz 5, A-8010 Graz, Austria

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# Preface

This volume contains the final versions of the invited lectures presented at the “Internationale Universitätswochen für Kern- und Teilchenphysik” in Schladming, Austria, February 1992, which was held under the topical title “Computational Methods in Field Theory”. A subject like the one discussed at our school would have been impossible at the first school in the series, 30 years ago. Field theory at that time was limited to analytic attempts for a solution such as Feynman graphs perturbation series; computational techniques were clever applications of the theory of special functions.

Today we have a wealth of new insight into problems in the context of quantum field theories gained by the computational approach. The first field to make heavy use of computer simulations for the solution of its problems was statistical physics. When the deep connections between the critical behaviour of spin models and the properties of regularized quantum field theories became clear, it was only a question of time and computer power before these computational techniques were applied to models in elementary-particle physics, starting in the late 1970s.

Meanwhile this field is becoming a discipline of its own. With the advent of powerful computers the spectrum of applications in physics has widened enormously and cannot be covered by any single conference or workshop. However, at our school in Schladming we always concentrated on problems in elementary-particle physics; thus we put the emphasis on those topics and methods in (quantum) field theory which have been particularly influenced by the computational approach.

John Klauder’s lecture reviews in a tutorial-like fashion the issue of random variables and stochastic processes in field theory, including the stochastic approach to field quantization, which has been utilized directly in simulations of bosonic and fermionic systems.

In Oscar Lanford’s lecture the impact of computers on mathematics and in particular theorem proving is analyzed. Emphasis is put on rigorous quantitative estimates.

In the process of applying the methods of statistical physics to the problems of gauge field theories one has to learn how to deal efficiently with the new types of systems. Computer studies are necessarily limited in (CPU) time and space (finite-size). Since one is interested in the behaviour of the system at large correlation lengths, finite-size effects and critical slowing down are important issues.

The lectures of both Ulli Wolff and Kurt Binder deal with these problems, the first concentrating on critical slowing down and efficiency of algorithms and the second with finite-size effects and their taming and utilization. For certain spin systems new algorithms have evolved, allowing an efficient simulation of bosonic theories. Gauge systems have an essentially non-local structure, and improvements of the simulation algorithms are hard to achieve. The close methodological interrelationship between particle physicists working in the lattice-field approach and solid-state physicists working with spin systems has proven useful in many instances. One example is the problem of learning about surface tension and bubble formation in systems with a first-order phase transition. This is one of the topics dealt with by Kurt Binder in his lecture, demonstrating how to extract information from the finite-size dependence of distribution functions.

Thomas DeGrand surveys the situation in lattice QCD, for particle physicists perhaps the most challenging topic. Early promises turned out to be too optimistic and recent years have been characterized by hard and detailed work aimed at understanding the limitations, aspects and prospects of this approach.

The microscopic theory may look very different on a macroscopic scale. The appearance of a theory may change with the scale of observation, as illuminated by the renormalization process. Gerhard Mack discusses ideas for identifying effective theories. His contribution is co-written by T. Kalkreuter, G. Palma and M. Speh and covers new ideas in methods (such as the multigrid approach) and conception.

Fred James reviews the impact of computers on high-energy physics experiments and gives an overview of design, running and analysis. He puts particular emphasis on the (superficially) most trivial and at the same time most subtle and important issue: how to get good random numbers. In the last few years there has been enormous progress and his lecture provides "computer field theorists" with a good update and background.

In his lectures at the school Alexander Migdal discussed attempts to study quantum gravity with help of computer models. Most of the work presented has been published elsewhere and he decided not to contribute to this volume.

At the winter school there were many excellent seminars closely related to the topics of the lectures. We cannot publish them in this collection but want at least to give credit to the speakers. Thus there is a list of seminar contributions at the end, and the authors may be contacted directly for related information and material.

The organizers of the school (the editors of this volume) wish to thank the lecturers for their excellent contributions. The school was supported generously by the Austrian Ministry of Science and Research, the Styrian Government and other sponsors. The contributions were prepared by the authors and written in  $\text{T}_\text{E}\text{X}$  or  $\text{L}_\text{A}\text{T}_\text{E}\text{X}$  with the help of Springer's macro packages. Mrs. E. Neuhold helped in producing the final version.

July 1992  
Graz, Austria

*H. Gausterer*  
*C.B. Lang*

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## Contributors

*Binder, K.*

Institut für Physik, Johannes Gutenberg Universität Mainz  
Staudinger Weg 7, D-6500 Mainz, Germany

*DeGrand, T.*

Department of Physics, University of Colorado  
Boulder, Colorado 80309, USA

*James, F.*

CERN, Data Handling Division  
CH-1211 Genève 23, Switzerland

*Klauder, J.R.*

Departments of Physics and Mathematics, University of Florida  
Gainesville, FL 32611, USA

*Lanford III, O.E.*

Mathematics Department, ETH-Zürich  
CH-8092 Zürich, Switzerland

*Mack, G., Kalkreuter, T., Palma, G., Speh, M.*

II. Institut für Theoretische Physik, Universität Hamburg,  
Luruper Chaussee 149, D-2000 Hamburg 50, Germany

*Wolff, U.*

CERN, Theory Division  
CH-1211 Genève 23, Switzerland