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Nonlinear Evolution Operators and Semigroups

Applications to Partial Differential Equations

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Preface.

The first aim of this book is to present in a coherent way some of the fundamental results and recent research on nonlinear evolution operators and semigroups. The second aim is to show how to apply these abstract results to unify the treatment of several types of partial differential equations arising in physics (the heat equation, wave equation, Schrödinger equation, and so on).

The motivation of this theory is clearly pointed out in the following quotation from: Autumn Course on Semigroups, Theory and Applications, held at the International Centre For Theoretical Physics, Trieste (Italy), 12 November - 14 December 1984 (Brezis-Crandall- Kappel, Directors).

"The last two decades have witnessed a tremendous use of semigroups and evolution equations techniques in solving problems related to PDE and FDE. This allows the treatment of PDE and FDE as suitable ODE in infinite dimensional Banach spaces. This method has considerably simplified and clarified the the proofs, and has unified the treatment of several different classes of differential equations. It has solved many problems that had been left open by previously known methods, and has been very succesful in dealing with discontinuous data and regularity."

Chapter 1 deals with the construction and main properties of nonlinear evolution operator $U(t, s)$ associated with a class of nonlinear (possible multivalued) operators $A(t)$ with time dependent domain, satisfying Hypotheses $H(2.1)$ and $H(2.2)$ in Section 2. We also say that $U(t, s)$ is associated with the nonautonomous differential equation (inclusion) $x'(t) \in A(t)x(t)$. In the convergence of DS-approximate solutions (i.e., in the construction of $U(t, s)$) the fundamental estimate is given by (2.40), essentially due to Kobayashi, Kobayasi and Oharu. Among other general results, we mention Theorem 5.1 which gives a characterization of the compactness of evolution operators.

Note that $U(t, s)$ associated with the equation $x'(t) \in A(t)x(t)$ allows a unifying treatment of the existence, uniqueness and behaviour of the various types of solutions to the Cauchy problem for this equation.

Chapter 2 is devoted to nonlinear semigroups $S_A(t)$ which are generated by the DS-limit solutions associated with the dissipative operator A. In the case A - m-dissipative, $S_A(t)$ is given by the exponential formula of Crandall-Liggett. We say also that $S_A(t)$ is generated by A via the exponential formula. The semigroup approach is important in the study of the solutions of the autonomous differential equation $x' \in Ax$, which includes several different classes of PDE and FDE.

In order to avoid duplication and to reduce the length of this work, we have tried to make (as much as possible) the autonomous case as a special subcase of the time-dependent case (this was also a suggestion of the referee). Of course this is an economic way to present such a theory, but not the simplest one. For the sake of simplicity, the reader may start with the autonomous case.

In the theory of the generation of nonlinear semigroups, the fundamental estimate (given by (1.16)) due to Kobayashi, is derived from (2.40) in Chapter 1, i.e., from nonautonomous case.

In Chapter 3, one applies the results of Chapters 1 and 2, both to a class of multivalued evolution equations and to some partial differential equations modelling physical phenomena.

Most of the results here are presented for the first time in a book (e.g., Brezis' characterization of nonlinear compact semigroups in Chapter 2, the theory of nonlinear evolution operators in Chapter 1 and most of the material in Chapter 3. Some of the results are very recent and not yet published (e.g., the characterization of compactness of evolution operators given by the author, the characterization of compactness of a linear semigroup solely in terms of the resolvent of its infinitesimal generator due to Vrabie and so on).

The discussions (at the "Al.I.Cuza" University of Iasi - Romania) with my colleagues Prof. V. Barbu, C. Ursescu and I. I. Vrabie have contributed to the improvement of many sections in this book. I am expressing my thanks to all of them.

Part of this work has been written during my long stay at the International Centre for Theoretical Physics (ICTP) and SISSA, Trieste (Italy). I am very grateful to Professor Abdus Salam, Nobel Laureate, founder and the Director of the ICTP, for the pleasant hospitality and stimulating discussions.

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